10. After one minute, a hot air balloon had risen 90 feet. Each succeeding minute, after that time, the balloon rose 70% as far as it did during the previous minute.

a. How far above the earth was the balloon after 8 minutes?

\[ 90 + 90(0.7) + 90(0.7)^2 + \cdots + (90)(0.7)^7 \]

\[ = \frac{90(1 - 0.7^8)}{1 - 0.7} \]

\[ = \frac{90(1 - 0.7^8)}{0.3} \]

\[ = 282.7 \]

b. What is the maximum height the balloon will rise?

\[ 90 + 90(0.7) + \cdots \]

\[ = \frac{90}{1 - 0.7} \]

\[ = 300 \text{ ft} \]

11. For what values of \( x \) will the series \( \sum_{k=0}^{\infty} (x - 3)^k \) have a finite sum?

\[ N_{\epsilon} \left| 1 + \frac{|x - 3|}{1 - |x - 3|} \right| < 1 \]

\[ = 1 - \frac{1}{1 - |x - 3|} < 1 \]

\[ = 2 < x < 4 \]
8. Vivian goes home for the weekend and her friend asks her what she is studying in math. Vivian says sequences like:

\[ 1, -60 \]

and so on. Her friend says that is not right because she knows sequences are lists of numbers like

\[ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \]

If you were Vivian, what would you say to convince her friend that she is correct? Your answer should demonstrate you understanding of the definition of a sequence.

A sequence is a function that assigns an element from some set, in Vivian's case, the set of all regular polygons to a subset of \( \mathbb{N} \). Here are some examples:

- 3 \( \rightarrow \) equilateral triangle
- 4 \( \rightarrow \) square
- 5 \( \rightarrow \) pentagon
- 6 \( \rightarrow \) ??

9. Evaluate each of the following.

3 a. \[ \sum_{k=8}^{50} (5-4k) = \frac{43}{8} (5 + 1) \]

\[ = \frac{43}{8} \cdot 38 \]

\[ = 195 \]

4 b. \[ \sum_{k=1}^{80} k^2 (k+2) = \sum_{k=1}^{80} k^2 + 2k^2 \]

\[ = \left[ \frac{80 \cdot (81)}{2} \right]^2 + 2 \cdot \frac{80 \cdot (81)(261)}{6} \]

\[ = \]
4. Find the sum of the arithmetic series $a_1 + a_2 + a_3 + \cdots + a_{52}$ given $a_2 = 12$ and $a_{52} = -138$.

$$d = \frac{-138 - 12}{52 - 2} = \frac{-150}{50} = -3$$

$$\Rightarrow a_1 = 12 + 3 = 15$$

$$S_n = \frac{n}{2} \left(2a_1 + (n-1)d\right)$$

$$= \frac{52}{2} \left(15 + (-138)\right)$$

3. Let $a_n = \{(x,y) | x = 2n \text{ and } y = 3n\}$. Write out the terms of $\{a_n\}_{n=2}^5$.

$$\{(4,6), (6,9), (8,12), \ldots\}$$

5. Given $S_n = 3n^2 - n$. Find $a_5$.

$$a_5 = S_5 - S_4$$

$$= 70 - 44$$

$$= -26$$

4. Express in sigma-notation: $3 - \frac{7}{4} + \frac{11}{9} - \frac{15}{16} + \cdots - \frac{79}{400}$

$$\sum_{k=1}^{20} \frac{(-1)^{k-1}(-1 + 4k)}{k^2}$$
You may use a TI-30 Calculator on this exam. Justify all your work.

Useful formulas:
\[
\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4}
\]

1. Given \( a_k = \begin{cases} 3, & \text{if } k = 1 \text{ or } k = 2 \\ 2a_{k-1} + a_{k-2}, & k > 1 \end{cases} \) and \( S_n = \sum_{k=1}^{n} a_k \)

a) State the first five terms of the sequence \( \{a_n\}_{n=1}^{4} \).

\[ \begin{align*}
3, 3, \text{ } a_3, \text{ } 21 \\
\end{align*} \]

b) State the first five terms of the sequence \( \{S_n\}_{n=1}^{4} \).

\[ \begin{align*}
3, \text{ } 6, \text{ } 15, \text{ } 36 \\
\end{align*} \]

2. Given a harmonic sequence with \( a_5 = \frac{15}{2} \) and \( a_8 = \frac{15}{11} \). Solve for \( a_{12} \). Give exact answer.

\[
\begin{align*}
&\frac{15}{2}, \text{ } \frac{15}{5}, \text{ } \frac{15}{11}, \text{ } \frac{15}{17}, \text{ } \frac{15}{20}, \text{ } \frac{15}{23}, \text{ } a_{10}, \text{ } a_{11}, \text{ } a_{12} \\
\end{align*}
\]

Denominators go up by 3

\( a_{12} = \frac{15}{23} \)

3. The eighth term in a geometric sequence is 10 and the twelfth term is \( \frac{5}{8} \). Find the explicit formula. Twenty-three terms.

\[
\begin{align*}
a_n &= 40 \left( \frac{1}{4} \right)^{n-1} \\
\end{align*}
\]

Explicit formula:

\[
\text{Explicit formula: } a_n = 40 \left( \frac{1}{4} \right)^{n-1} \]