Show enough work that I can reproduce your results.

No Calculator allowed.

1. (2 pts each) For each sequence given below, mark which type(s) of sequence it could be:

   a. \[ \frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \ldots \]  
      \( \lambda = \frac{1}{2} \)  
      - Arithmetic  
      - Geometric  
      - Harmonic  
      - None of these

   b. \[ \frac{-4}{1'}, \frac{5}{4'}, \frac{6}{9'}, \frac{7}{16'}, \frac{8}{25'}, \ldots \]  
      - Arithmetic  
      - Geometric  
      - Harmonic  
      - None of these

   c. \[ 3, 3, 3, \ldots \]  
      - Arithmetic  
      - Geometric  
      - Harmonic  
      - None of these

   d. \[ \frac{5}{12'}, \frac{-5}{6'}, \frac{5}{3'}, \frac{-10}{3'}, \ldots \]  
      - Arithmetic  
      - Geometric  
      - Harmonic  
      - None of these

2. (4 pts) Find the eighth term of the geometric sequence for which \( a_2 = 10 \) and \( a_5 = 20\sqrt{2} \).

   \[ a_5 = a_1 r^3 \]  
   \[ 20\sqrt{2} = 10 r^3 \]  
   \[ r = \sqrt[3]{2} \]  
   \[ a_1 = \frac{10}{\sqrt[3]{2}} = 5\sqrt[3]{2} \]  
   \[ a_n = 5\sqrt[3]{2} \cdot (\sqrt[3]{2})^{n-1} \]

3. (4 pts) Given that the sequence below is a harmonic sequence, write the next two terms, then write an explicit formula for the sequence.

\[ a_n = 6, 2, \frac{6}{5}, \frac{6}{7}, \ldots \]  
\[ b_n : \frac{1}{6}, \frac{1}{4} , \ldots \]  
\[ d = \frac{1}{3} \]  
\[ b_n = \frac{1}{6} + \frac{1}{3}(n-1) \]  
\[ a_n = \frac{6}{2n-1} \]  

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4. (3 pts) Write out the terms of the sequence \( \{a_n\}_{n=1}^{4} \) where \( a_n = \begin{cases} 1 & \text{if } n = 1 \\ \frac{1}{1 + a_{n-1}} & \text{if } n > 1 \end{cases} \)

\[
\begin{align*}
a_1 &= 1 \\
a_2 &= \frac{1}{1 + \frac{1}{2}} = \frac{2}{3} \\
a_3 &= \frac{1}{1 + \frac{2}{3}} = \frac{3}{5} \\
a_4 &= \frac{1}{1 + \frac{3}{5}} = \frac{5}{8}
\end{align*}
\]

5. (5 pts) Find the explicit and recursive formulas for the arithmetic sequence with \( a_9 = 20 \) and \( a_{30} = 83 \).

Explicit:
\[a_n = -4 + 3(n - 1)\]
\[a_n = 3n - 7\]

Recursive:
\[\begin{align*}
a_1 &= 20 \\
da &= \frac{83 - 20}{30 - 9} = \frac{63}{21} = 3 \\
a_n &= \begin{cases} -4 & \text{if } n = 1 \\ a_{n-1} + 3 & \text{if } n > 1 \end{cases}
\end{align*}\]

6. (2 pts extra credit) Given that \( x^2 - 10, x, \frac{2x}{x-1} \) is a geometric sequence, solve for \( x \).

\[
\begin{align*}
r &= \frac{x}{x^2 - 10} = \frac{\frac{2x}{x-1}}{x} \\
x(x-1) &= 2(x^2 - 10) \\
x^2 - x &= 2x^2 - 20 \\
0 &= x^2 + x - 20 \\
(x+5)(x-4) &= 0 \\
x &= -5 \text{ or } x = 4
\end{align*}
\]

\[
\begin{align*}
x &= -5 \quad & \text{or} & \quad x &= 4 \\
15, -5, \frac{5}{3} \\
6, 4, \frac{8}{3}
\end{align*}
\]