Practice Quiz

Clearly show ALL appropriate work for full credit. NO magically calculator leaps of faith.

1. Find an upper bound for the error made in approximating \( \cos(3.5) \) using the fourth degree Maclaurin series. Round your final answer to 5 decimal places.

\[
\left| \cos(3.5) - P_4(3.5) \right| \leq \frac{(3.5)^6}{6!} = \frac{(3.5)^6}{2.55}
\]

This is not such a good approx.

2. When approximating \( e^{\frac{4}{3}} \), how many nonzero terms of the Maclaurin series for \( f(x) = e^x \) do you have to use in order to have an error that is less than 0.00001? No fair using the \( e^x \) key when solving this problem, so just remember that \( 2.5 < e < 3 \). (4 pts)

\[
e^{\frac{4}{3}} = 1 + \left( \frac{4}{3} \right) + \frac{\left( \frac{4}{3} \right)^2}{2} + \cdots + \frac{\left( \frac{4}{3} \right)^n}{n!} + R_n \left( \frac{4}{3} \right)
\]

\[
\text{Error} = \left| \frac{f^{n+1}(c)}{(n+1)!} \left( \frac{4}{3} \right)^{n+1} \right|
\]

\[
= \frac{e^c}{(n+1)!} \left( \frac{4}{3} \right)^{n+1} \quad \text{where} \quad \frac{4}{3} < c < \frac{4}{3}
\]

\[
\leq \frac{9}{(n+1)!} \left( \frac{4}{3} \right)^{n+1}
\]

Solving \( \frac{9}{(n+1)!} \left( \frac{4}{3} \right)^{n+1} \leq 0.00001 \) for \( n \)

\[
\Rightarrow \quad n > 9
\]

Take \( n = 11 \) nonzero terms

\[
\frac{1}{10^{11}} \text{ degree polynomial}
\]