Singular Systems

Learning Goals: Students see the difference between dependent, inconsistent, and non-singular systems

Some of the systems we wrote down had exactly one solution. Some had many. Others had none. Ideally there is exactly one solution. This is called the non-singular case.

Singular systems are broken down into two types:
- Inconsistent systems have no solutions
- Dependent systems have more than one solution

Inconsistent systems

In an inconsistent system there is no solution. The simplest such system is the single equation $0x = 1$ (or any other non-zero number). What happens in an inconsistent system?
- Row picture: the equation’s solution sets have an empty intersection. This could occur because you have parallel lines (system 3) or too many lines that don’t all meet in a common place (system 6)
- Column picture: the right hand side is not a linear combination of the vectors on the left hand side. For instance, in system 6, the two three-dimensional vectors have linear combinations that can fill up a plane, but the right-hand side is not on this plane

Dependent systems

In a dependent system there are many solutions. The simplest such system is the single equation $0x = 0$, for which any number is a solution. What is the geometry of dependent systems?
- Row picture: the intersections of the solution sets don’t get down to a single point. For instance, you might have two equations of planes in space. The intersection of two planes is either empty (parallel planes, giving an inconsistent system) or an entire line, all of which are solutions to both equations.
- Column picture: there is more than one way to combine the vectors to get the desired result. This is like having different sets of directions lead to the same place (i.e., either go one block north and one block west, or go northwest).