So what is a linear equation? It is an equation in which no variable shows up to any power or multiplied by another variable, and in which no functions like sine or logarithm appear. There is a technical definition which we don’t need, because we really know what they are.

We will mostly study systems of linear equations, which are one or more linear equations which are to be solved simultaneously. Some examples:

(1) \( 2x + 3y = 7 \)
(2) \[
\begin{align*}
3s - 2t &= 5 \\
6s + t &= -10
\end{align*}
\]
(3) \[
\begin{align*}
2x + 4y &= 6 \\
3x + 6y &= 8
\end{align*}
\]
(4) Five coins (nickels, dimes, and quarters) total fifty cents—leads to
\[
\begin{align*}
n + d + q &= 5 \\
5n + 10d + 25q &= 25
\end{align*}
\]
(5) There are 300 people on an airplane, including three times as many men as women—
\[
\begin{align*}
m + w &= 300 \\
m &= 3w \text{ (or } m - 3w = 0) \]
\]
(6) Find a line through the points (1, 2), (2, 4), and (4, 5), which gives the equations
\[
\begin{align*}
2 &= m + b \\
4 &= 2m + b \\
5 &= 4m + b
\end{align*}
\]

Note that we usually will write systems as in (4), (5), or (6), not including the brace as in (2) or (3).

Now that we know what linear systems are, how do we solve them? First, we need to know what solutions are. A solution to a system is an assignment of variables that makes all of the equations true. There may be more than one solution, and there may be none at all. In the examples above:

(1) has (2, 1), (–1, 3), and (11, –5) as solutions, but not (4, 4) or (1, –1)
(2) has (–1, –4) as its unique solution
(3) has no solutions at all
(4) has (0, 5, 0) and (3, 1, 1) as solutions. It also has (6, –3, 2) and (3/2, 3, 1/2) as solutions, through they don’t make any sense in context.
(5) has 225 men, 75 women as its only solution.
(6) has no solutions

Occasionally, as in (4) and (5) there will be context which we have to match, but that will not be our primary worry.