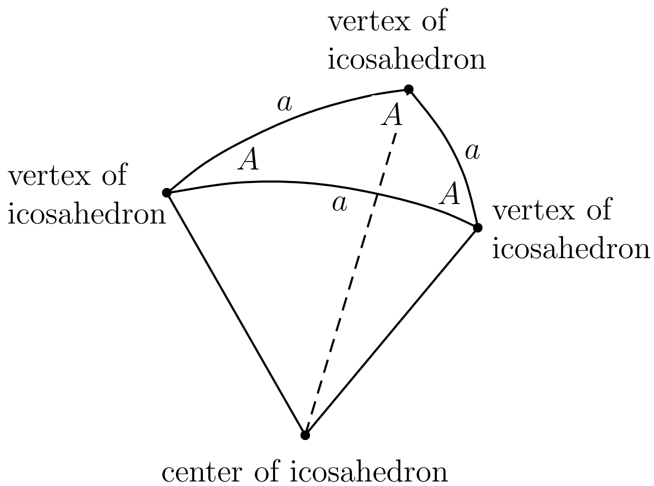
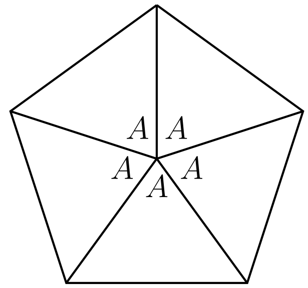
Now that you understand just *what* an edge angle is, how would you go about finding one?



Consider the figure above, where a face of an icosahedron is projected onto a sphere to form a spherical triangle. Note that a is the edge angle of the icosahedron. Using spherical trigonometry, find an expression for $\cos a$ in terms of A.

Now what is A? Imagine our icosahedron projected onto a sphere, and look down at a vertex and see the five spherical triangles adjacent there, as in the figure below.



What must A be? Briefly explain your answer.

Use the value you found for A to find $\cos a$. Simplify your answer as much as possible. Your final result should look like

$$\cos a = \frac{1}{\sqrt{k}},$$

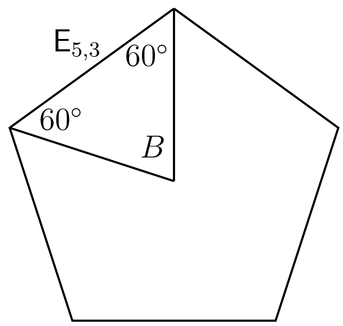
where k is an integer.

For which other Platonic solids can we apply the same procedure and find the edge angle? Do it now!

You have just found the edge angles for the three Platonic solids with triangular faces. We use the notation $\mathsf{E}_{p,q}$ for the edge angle of the Platonic solid $\{p, q\}$, so that you have just found $\mathsf{E}_{3,3}$, $\mathsf{E}_{3,4}$, and $\mathsf{E}_{3,5}$.

What about $\mathsf{E}_{5,3}$ and $\mathsf{E}_{4,3}$? Let's look at the dodecahedron first. It helps to imagine a dodecahedron projected onto a sphere, making twelve spherical pentagons. Now focus on a particular face, and divide this pentagon into five congruent spherical triangles meeting at its center.

One such triangle is illustrated below.



Why do the indicated angles have measure 60°? Provide a brief explanation.

What is the measure of B? Justify your answer.

Now use spherical trigonometry to find the edge angle of the dodecahedron. Simplify so that your final result is in the form

$$\cos \mathsf{E}_{5,3} = \frac{\sqrt{r}}{s}.$$

Now use this same method to find the edge angle of the cube.