We're going to embark on the task of deriving one of the most fundamental formulas of spherical trigonometry. We'll use it over and over and over again.... Of course you recall a famous Euclidean formula, where a, b, and c are sides of a triangle and C is the angle opposite c:

$$c^2 = a^2 + b^2 - 2ab\cos C.$$

Begin by cutting out and assembling the "spherical triangle" given to you with this worksheet. It's a representation that we can work with and that will help us visualize all six angles of a spherical triangle. Note that p, q, and r would be the vertices of the actual spherical triangle, but we'll work with p', q', and r'.

Note that a, b, and c are the edge angles of the triangle, and the angles at the folds of the paper are the vertex angles. We're particularly interested in the vertex angle C made at the fold between edge angles a and b.

Now we're going to label some of the edges made *on the model.* So write "1" near the segment Op'; we're going to use that side as the unit length. Now, using your knowledge of plane trigonometry, label the segments $p'q'$, Oq', $p'r'$, and Or' in terms of trigonometric functions of a, b, and c. Write down their lengths below.

Now let's look at $[q'r']^2$ in two different ways. First, consider $\Delta Oq'r'$, and write the cosine law for plane triangles with $c = [q'r']$. Use the lengths of the segments you've written down on your model.

Now consider $\Delta p'q'r'$. Since $p'q'$ is perpendicular to Op', $\angle q'p'r'$ must actually be the vertex angle C (opposite side c) of the spherical triangle. Study the model for a few minutes to see why this must be the case. With this in mind, write *another* expression for $[q'r']^2$ by considering $\Delta p'q'r'$.

Now set these two expressions for $[q'r']^2$ equal to each other. Using some trigonometric identities and doing a little algebra, solve for $\cos c$. You've just discovered your first (and most important!) spherical trigonometry formula. Good work!