Everything You've Always Wanted to Know About Trigonometry...

We're going to need trigonometry. Lots of it. So unless you've remembered everything you ever learned about trigonometry, let's start reviewing. First, write the definitions of $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$ in terms of $\sin\theta$ and $\cos\theta$.

Recall that a function f of θ is **even** if $f(-\theta) = f(\theta)$, and **odd** if $f(-\theta) = -f(\theta)$. For each of the six trigonometric function, determine whether it is even or odd. Drawing a unit circle might help you.

Defining the trigonometric functions in terms of the unit circle yields the fundamental

$$\cos^2\theta + \sin^2\theta = 1.$$

Divide this equation first by $\cos^2\theta$ to obtain another identity, and then by $\sin^2\theta$ to obtain another. (Assume that both $\cos\theta \neq 0$ and $\sin\theta \neq 0$.)

By considering a right triangle, evaluate the following:

$$\sin\left(\frac{\pi}{2} - \theta\right) = \qquad \cos\left(\frac{\pi}{2} - \theta\right) =$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \qquad \cot\left(\frac{\pi}{2} - \theta\right) =$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \qquad \csc\left(\frac{\pi}{2} - \theta\right) =$$

Recall the ever-important

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

Using the fact that

$$\sin(\alpha + \beta) = \cos\left(\frac{\pi}{2} - (\alpha + \beta\right),$$

derive an expression for $\sin(\alpha + \beta)$.

Writing

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)},$$

find a formula for $\tan(\alpha+\beta)$ involving *only* $\tan\alpha$ and $\tan\beta$. Do the same for $\tan(\alpha-\beta)$.

By writing

$$\cos\theta = \cos\left(\frac{\theta}{2}+\frac{\theta}{2}\right)$$

derive the usual "half-angle" identities.

Now use these identities to find *two* different formulas for $\tan\dfrac{\theta}{2}$. Study the "signs" (+ or −) in front of the expressions you derived and determine in which quadrant θ must lie for each of the signs to apply.

Finally, using a method suggested by the figure below, show that the area of the triangle is $\dfrac{1}{2}\,ab\,\sin C$.