

Mathematical Investigations

Quadratic Parameters

Purposes:

To examine the effects of parameters in the equation $y = ax^2 + bx + c$. We keep two parameters constant while varying the third, rotating through all cases. The constants which have been chosen are not important by themselves, and neither are the specific resulting patterns. What becomes noteworthy is the fact that such patterns do exist. This activity sheet will push students to practice graphing parabolas, finding vertices, and create functions which join these vertices.

Prerequisites:

- (1) Students should be familiar with quadratic functions and their graphs.
- (2) Students should be able to find the vertex of a parabola by at least one method.
- (3) Students may wish to use linear and/or quadratic regression on their calculator to find the resulting patterns.
- (4) The technique of completing the square will be useful to prove relationships between vertices that they have found.

Notes:

This activity will take some time and patience among students. Working in groups and sharing results will help alleviate these issues a great deal. Even so, students will need to be careful in their work. However, students usually realize fairly quickly when a vertex needs to be corrected, so this will not be a big problem. Students can often find the pattern, particularly if they resort to regression, but proving that is another matter entirely. Students should be encouraged to work on this with other students first, but going through this together in class will often be necessary to prove the general case.

Technology is wonderful for showing these relationships. On a Texas Instruments calculator, one can enter, for example, $y1 = \{-4, -2, 0, 2, 4\}x^2 + 4x + 6$ to see several graphs at once. A nicer approach, if you have access to a computer, is to create an animation. Consider a simple program such as *Graphing Calculator* (Mac) or *WinPlot* (Windows) to do elegant animations easily. With more prerequisites for use, try *Mathematica*, *Maple* or another computer algebra system. All of these can give a dynamic View of the effects of the parameters.

Mathematical Investigations
Quadratic Parameters

We want to consider the general quadratic function, $f(x) = ax^2 + bx + c$, and what happens to the graph of the function as the parameters a , b , and c change. Specifically, let's look at each parameter in turn and how it affects where the vertex of the parabola lies. Work together in groups of 3 or 4. One person take the first, another the second, and so on. Complete the square to find vertices of the 12 parabolas and share your results to complete the table:

1. Consider the quadratic function $y = x^2 + 4x + C$.

Your goal is to determine how its is graph affected by changes in C .

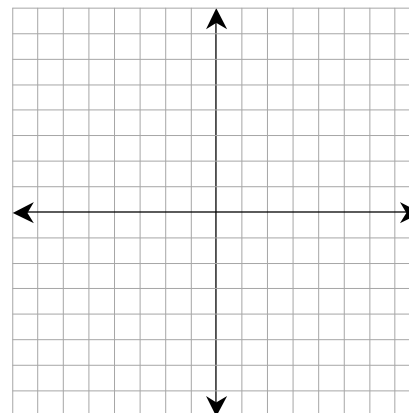
a. For each of the following values of C , find the vertex of the parabola.

C:	-5	-4	-3	-2	-1	0
(x, y):	(,)	(,)	(,)	(,)	(,)	(,)

C:	1	2	3	4	5	6
(x, y):	(,)	(,)	(,)	(,)	(,)	(,)

(Work space to show your work completing the square on the 3 or 4 quadratic functions you did)

b. Plot the twelve pairs of vertex coordinates on a new graph and derive the equation of the curve that would best fit those points. This is the "locus" of the vertices of the function $y = x^2 + 4x + C$ as C varies.



c. Describe, using complete sentences, how changing the parameter C affects the graph of the function $y = x^2 + 4x + C$.

2. Consider the quadratic function $y = x^2 + Bx + 6$.
Your goal is to determine how its graph is affected by changes in B .

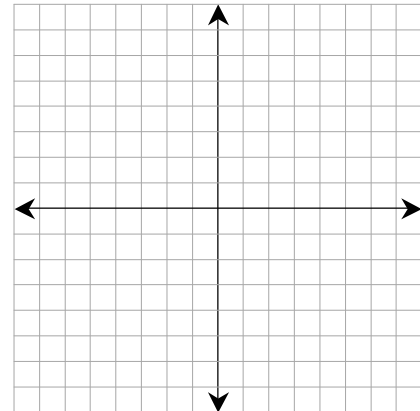
a. For each of the following values of B , find the vertex of the parabola. Again, divide the work among 3 or 4 of you and record the group results on your paper.

B : -10 -8 -6 -4 -2 0
 (x, y) : (,) (,) (,) (,) (,) (,)

B : 2 4 6 8 10 12
 (x, y) : (,) (,) (,) (,) (,) (,)

(Work space to show your work completing the square on the 3 or 4 quadratic functions you did)

b. Plot the vertex coordinates on a new graph and find the equation of the curve that would best fit those points.
This is the "locus" of the vertices of the function:
 $y = x^2 + Bx + 6$ as B varies.



c. Describe, using complete sentences, how changing the parameter B affects the graph of the function $y = x^2 + Bx + 6$.

3. Finally, consider the quadratic function $y = Ax^2 + 4x + 6$.
Your goal is to determine how is its graph affected by changes in A .

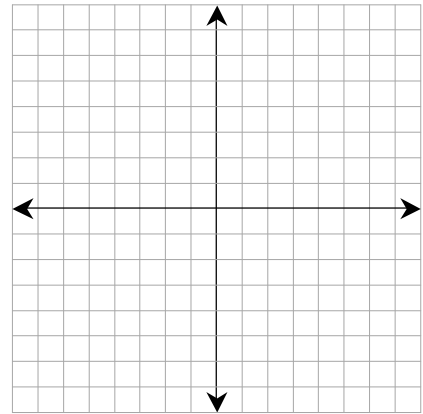
a. For each of the following values of A , find the vertex of the parabola. work together with your group to spread the work. It may be necessary to approximate coordinates in your graph.

A : -5 -4 -3 -2 -1 0
 (x, y) : (,) (,) (,) (,) (,) (,)

A : 1 2 3 4 5 6
 (x, y) : (,) (,) (,) (,) (,) (,)

(Work space to show your work completing the square on the 3 or 4 quadratic functions you did)

b. Plot the vertex coordinates on a new graph and find the equation of the curve that would best fit those points.



c. Describe, using complete sentences, how changing the parameter A affects the graph of the function $y = Ax^2 + 4x + 6$.