

Mathematical Investigations

Power Functions

Purpose:

To introduce graphs of power functions as a beginning to the study of polynomials. We start with the simplest of examples of polynomials, namely functions of the form $f(x) = x^n$, where n is a positive integer. We want students to become comfortable with the graphs and basic properties and patterns of such functions before continuing with more complex equations and graphs.

Prerequisites:

- (1) Students should be sufficiently familiar with their calculators to graph functions and then to label points on paper.
- (2) Students should be familiar with the concept of inverse functions and what conditions are necessary for a function to have an inverse which is also a function.

Notes:

Deciding where to put the axes and choosing an appropriate scale for each axis will often be difficult for students. One box for each unit will not work well to see the graphs clearly, particularly around the origin. Five graphs at once may be difficult to show, even for the most careful. Still, enough should be shown so that the pattern created by the exponents becomes clear. Problems (2) and (9) both ask for five points which lie on the graphs. We want to be sure that students can draw the graphs without a calculator, but the calculator will probably be very convenient for the last couple of points in each case.

A related activity sheet deals with the reciprocals of power functions (i.e. power functions with negative exponents).

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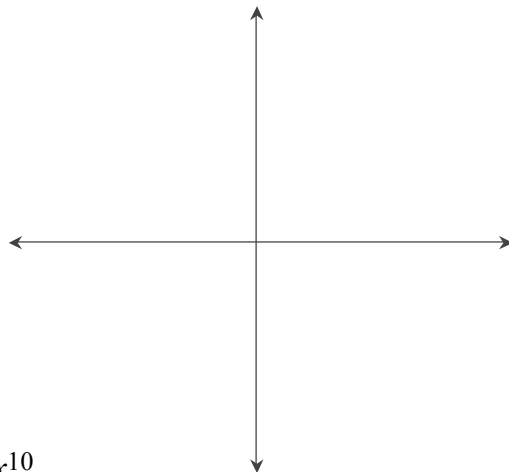
1. Sketch the graphs of each of the following functions. Choose a scale that allows you to distinguish each function. Label three points on each graph.

$$y_1 = x^2$$

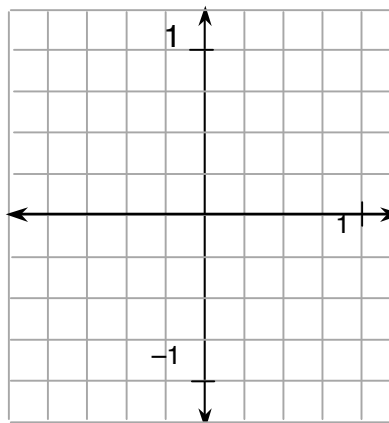
$$y_2 = x^4$$

$$y_3 = x^6$$

$$y_4 = x^8$$



2. **Without** using your calculator, add the graph of $y_5 = x^{10}$ to the above graph and list below five points that lie on the graph.
3. What points do all of these graphs have in common?
4. In what way(s) are the graphs different?
5. Does $y = x^0$ fit this pattern? Why or why not?
6. Do any of these functions have inverses that are functions? Why or why not?
7. Give a general description of the graph of $y = x^n$, where n is a positive, even integer.
8. a. Which graph, $y = x^2$ or $y = x^4$, is lower (flatter) in the window: $-1 \leq x \leq 1, -1 \leq y \leq 1$? (Sketch them to the right.) Why does this occur?
- b. Which graph will be "flatter" within one unit of the x-intercept: $y = x^4$ or $y = x^6$? Explain.



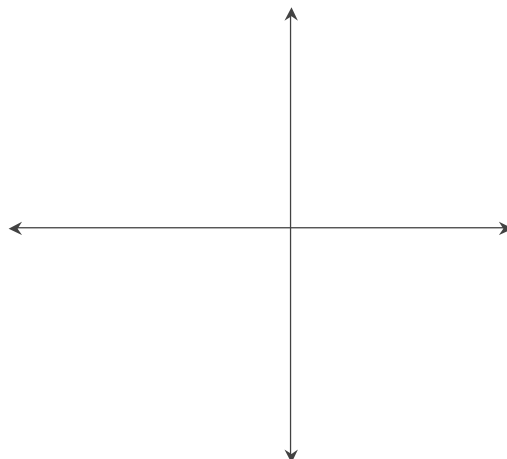
9. Sketch the graphs of each of the following functions using a scale that allows you to distinguish each function. Label three points on each graph.

$$y_1 = x^1$$

$$y_2 = x^3$$

$$y_3 = x^5$$

$$y_4 = x^7$$



10. **Without** using your calculator, add the graph of $y_5 = x^9$ to the above graph and list below five points that lie on the graph.

11. What points do all of these graphs have in common ?

12. In what way(s) are the graphs different?

13. Do any of these functions have inverses that are functions? What is the inverse?

14. Give a general description of the graph of $y = x^n$, where n is a positive odd integer.

15. a. Which graph, $y = x^1$ or $y = x^3$ or $y = x^5$, is lower (flatter) in the window: $-1 \leq x \leq 1, -1 \leq y \leq 1$? (Sketch them to the right.) Why does this occur?

- b. Which graph will be "flatter" within one unit of the x-intercept: $y = x^5$ or $y = x^9$? Explain.

