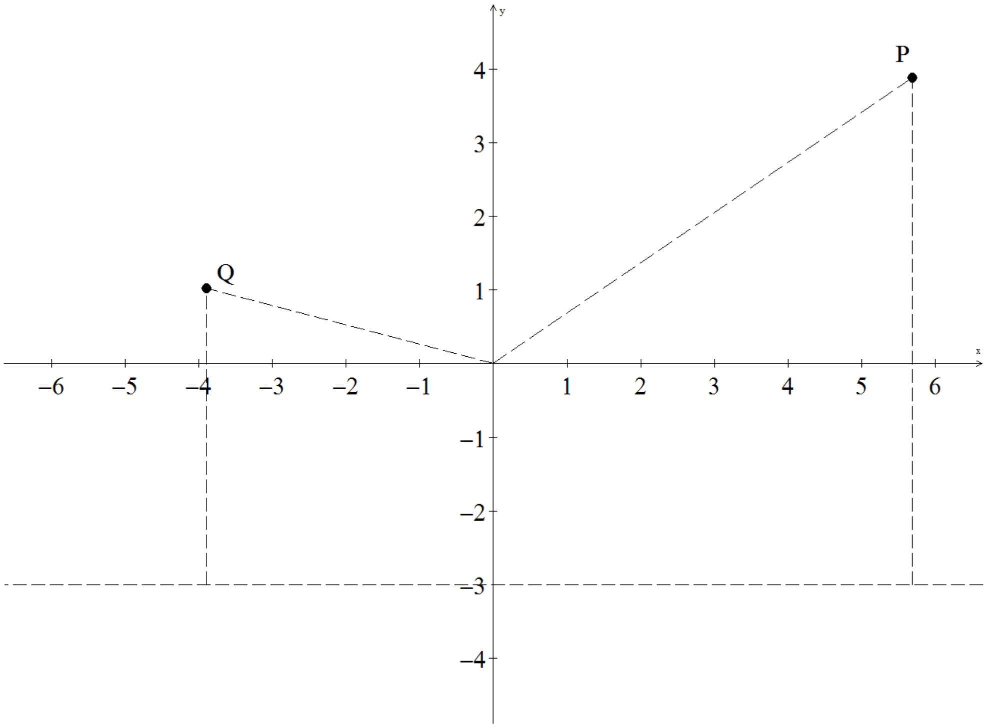

1) Consider the set of all points *P* equidistant from the pole and the horizontal line $y = -3$. The line is called a directrix and the pole is called the focus of this curve, a parabola

a) On your answer sheet, draw the locus of points that meets these conditions. *P* and *Q* are points on this graph.

b) Using polar variables *r* and θ, write an equation that says the distance from the *P* to the directrix equals the distance from *P* to the origin.

c) Rearrange (show steps) your equation so that it becomes $r = \dfrac{3}{1 - \sin\theta}$.

d) Using Winplot or another computer grapher, graph this curve and attach it to you problem set.

2) $\begin{bmatrix} \cos 25° & -\sin 25° \\ \sin 25° & \cos 25° \end{bmatrix} \cdot \begin{bmatrix} \cos 15° & -\sin 15° \\ \sin 15° & \cos 15° \end{bmatrix} = \begin{bmatrix} \cos 40° & -\sin 40° \\ \sin 40° & \cos 40° \end{bmatrix}$

a) Verify this result using trigonometry and the properties of matrices.

b) Explain why this result should have been expected in light of your previous unit and transformations.

3) Find a vector 5 units long in the direction of $\vec{u} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

4) (Calc OK) Let $T = \begin{bmatrix} 2 & 4 & -5 \\ 6 & 1 & -1 \end{bmatrix}$ be the vertex matrix for a triangle and let $A = \begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$ be a matrix.

a) Find the matrices $U = AT$ and $V = AU$.

b) Find the areas of the triangles whose vertex matrices are T, U, and V. (OK to use Shoelace.)

c) Find the determinant of A and make a conjecture about areas and determinants.

5) Show $\sum_{i=0}^{5} \binom{5}{i} \left(\frac{1}{3}\right)^{5-i} \left(\frac{2}{3}\right)^{i} = 1$ (Looks like something to do with the Naomi and Shawon problem from before.)

6) Find each sum:

a. $\sum_{k=0}^{\infty} \frac{4 - 2^{k+1}}{4^k}$

b. $24 + 20 + \frac{50}{3} + \frac{125}{9} + \cdots$

7) Find the exact value of $\sin\left(2\sin^{-1}\left(\frac{1}{4}\right)\right)$.

8) Solve the following for x, $0 \le x < 2\pi$:

$$\frac{\sin x}{1 + \cos x} = 1$$

9) Dr. Condie's daughter Natalie loves to chase their cats (Ella and Billie) around. Suppose Natalie is running directly northwest at 14 mph chasing Ella. She sees Billie out of the corner of her eye, and with an overwhelming desire to entertain Billie, she throws her ball while running directly at the cat, in the direction N30°E at a speed of 43 mph. Ignoring air resistance, what is resultant direction and speed of the ball? (P.S. She missed Billie ☹)

10) Find the rectangular equation of the parabola you graphed in problem #1.

11) The parametric form of the rectangular equation $y = x^2$ is $\begin{cases} x = t \\ y = t^2 \end{cases}$. You can convince yourself of this by removing the parameter t. Use a computer to graph the parametric form of this parabola and attach it to your answer sheets.

General Rotation Formula for Parametric Equations

The rotation of any parametric equations through an angle θ is given by

$$\begin{cases} x = x'\cos\theta - y'\sin\theta \\ y = x'\sin\theta + y'\cos\theta \end{cases}$$

where x' is the original equation for *x* and y' the original equation for *y*.

These formulas came from multiplying the rotation matrix $R_\theta \cdot \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$

12) Find the parametric equations of the rotation of $\frac{\pi}{4}$ of the parametric equations you graphed in problem #11. Plot this on the same grid as problem #11.

13) There is a theorem in mathematics that states any conic section can be written

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

In the case of $y = x^2$, $A = 1$, $E = -1$, and all other coefficients are zero.

Find the rectangular equation of the 45° rotation of the parabola you graphed in problem #12.

Note: Calculators OK but show enough work to follow your reasoning.

14) Exploration: A polar problem

When you first studied trigonometry, you explored $y = \cos\ bx$ and considered the effect of *b* on the graph of cosine. Consider the $r = 3\cos\left(\frac{a}{b}\cdot\vartheta\right)$ where *a* and *b* are integers. Explore the effect *a* and *b* have on the graph and summarize your results. Include several graphs supporting your conclusions. Your explanations should be in complete sentences that a fellow student that did not do the exploration can follow.