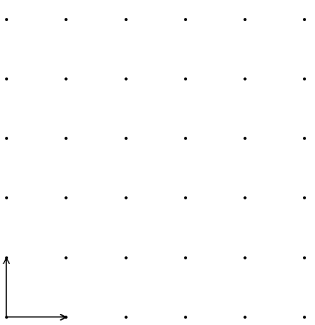
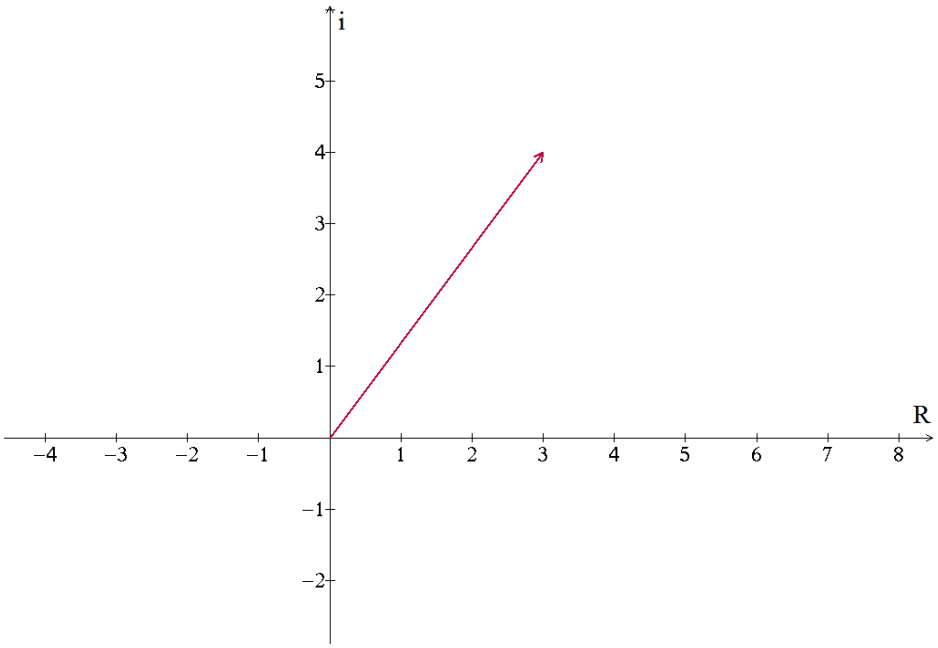
1) Naomi goes for one last walk. This time she starts from home (0, 0) and flips a coin at each corner to decide if she goes right or up. She decides a heads will be up and a tails right. Unbeknownst to her, her friend Shawon gives her a coin that comes up heads 25% of the time. If she walks five blocks, what is the probability she arrives at (2, 3)?



2) **NC** $\sum_{n=0}^{2011} \cos\left(\frac{n\pi}{4}\right)$ State your answer exactly.

3) We can think of numbers in the complex plane like they are vectors. The complex number $3+4i$ when graphed in the complex plane looks like this:



a) Find the measure (in degrees to 2 decimal places) of the angle between the complex numbers $-7+5i$ and $1+3i$.

b) Find the magnitude and the angle $4+3i$ makes to the *x*-axis. We will call this angle a **polar angle**.

4) Find the measure of the angle between the given vectors.

a) $\langle -7,5\rangle$ and $\langle 1,3\rangle$

b) $\langle 4,3\rangle$ and $\langle 1,0\rangle$

5) Solve for $0 \le x < 2\pi$: $2\cos^2 x + 3\sin x - 1 = 0$.

6) Solve for θ, where $-90° \le \theta \le 90°$:

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 4 \\ -5 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

7) A motor boat traveling South East 4 m/s encounters the river current moving North 6 m/s.

a. What is the resultant velocity of the motorboat?

b. If the river width is 90 meters, calculate the time the takes to travel shore to shore.

c. What distance downstream does the motorboat reach the opposite shore?

8) Jack opens a savings account with a $100 deposit at the beginning of the month. The account earn 5.6% annual interest compounded monthly. At the beginning of each subsequent month, Jack deposits an additional $100. How much will the account be worth at the end of 10 years? [Hint: This is a geometric series.]

9) Somewhere back in MI 3, you found $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ approached a constant, $2.718\ldots$ so we ended up defining this number to be the mathematical constant *e*. In this problem you will consider $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n$.

a) Replace *n* with $2k$ in the above limit and write on your answer sheet.

b) Simplify this expression and write it in terms of *e*.

c) Generalize your result by expressing $\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n$ in terms of *e*. You could check this with a BC 3 student to confirm your result.

10) A fishing boat leaves port heading 70° (on the compass) at 8 mph for 3 hours, then turns to a heading of 300° at 10 mph for 2 hours, and finally changes to a heading of 30° at 6 mph for 4 hours. From this point the boat heads directly back to port at 10 mph. Find the boats heading as it returns to port, and the time it takes to return to port.

11) $\vec{V} = \langle -3, 4 \rangle$ and $\vec{W} = \langle -3, 8 \rangle$

a) $\text{Proj}_W \vec{V} =$

b) $\left| 2\vec{V} + 3\vec{W} \right| =$

c) Find a such that $\langle 4, a \rangle \perp \vec{W} - \vec{V}$

d) Find a unit vector that makes a $45°$ angle with $\vec{V}$. OK to use calculator to solve any system.

12) $1 + (x-1) + (x-1)^2 + (x-1)^3 + (x-1)^4 + \ldots = \frac{3}{4}$. Find x.

13) Use your CAS calculator to do this problem with complex mode set to rectangular. For each of the following, state the result in standard $(a + bi)$ form of a complex number (using three decimal places) and find the magnitude and the polar angle (see problem 3) for each of the following. State your angle in radian measure, to three decimal places.

a) $\left(1 + \frac{i}{2}\right)^2$

b) $\left(1 + \frac{i}{3}\right)^3$

c) $\left(1 + \frac{i}{10}\right)^{10}$

d) $\left(1 + \frac{i}{100}\right)^{100}$

Something interesting is going on here, especially if you consider what is behind problem #9.

14) Ethan is a curious fellow. He was playing (investigating) his TI-89 and set its mode to complex format 'polar' then typed in the complex number in 3b and presses 'diamond enter'.

a) What results did Ethan get? (Hmm, now problem 13 is even more interesting.) Give two answers for this, first in radian mode, then degree mode.

Now Ethan is really curious and he thinks he has a shortcut to find the polar angle and magnitude of any complex number. He wonders what will happen if he raises the number to a power.

b) Find the magnitude and polar angle of $(3+4i)^3$.

15) Exploration. Explore the effect on the magnitude and polar angle of a complex number when it is raised to a power (See #14). For this exploration Madeline and Robin would suggest to first choose a complex number of magnitude 1 and see what happens, then examine other complex numbers. On your answer sheet write your conclusions with two examples to demonstrate your conjectures. Write your explanations in complete sentences so a fellow student like Ethan could follow your logic.