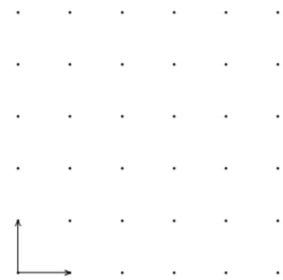
IMSA **MI-4** Prob. Set **#8** **DUE**: Thursday, April 14 Spring '11

1) Recall problem 1 from problem set 7. This time Naomi starts from home (point (0, 0)) and flips a balanced coin to determine which way to walk, up or to the right. She plans to walk four blocks, but her path is determined by the coin toss.



a) List the coordinates of all the points that she may end here walk, and for each point, list the number of routes that gets her there. For example, she can arrive at point (3, 1) by these routes: RRRU, RRUR, RURR, and URRR, so there are four routes ending pint (3,1).

b) For each point listed in part (a), state the probability she arrives at each point.

2) Monica stepped outside and began to think how fast she is moving even when she is standing still. She recalled from previous work that a rotational speed translates into a linear speed. Assume the earth is a sphere with a radius of 3960 miles and Monica is at $41°$ N latitude.

a) What is Monica's speed (in mph) due to the rotation of the earth?

b) Assume the earth rotates around the sun in a circle of radius 93,000,000 miles. What is Monica's speed (in mph) just due to the rotation of the earth about the sun? Use 365.25 days in a year.

c) Combining the two, give the range of speeds Monica is moving while she is standing still. Explain.

3) Find the exact value of $\cos\left(\sin^{-1}\left(\frac{12}{13}\right)+\tan^{-1}\left(\frac{4}{3}\right)\right)$.

4) Solve the following equations for $0 \le x < 2\pi$, giving **exact** values.

$$\sqrt{3}\sin 2x - \cos 2x = 2$$

5) **NC** Simplify each trigonometric expression:

a. $\frac{\tan^2 \mu}{\sec^2 \mu}+\frac{\cot^2 \mu}{\csc^2 \mu}$

b. $\cot \alpha \sin 2\alpha - \cos 2\alpha$

6) Find constants A, B, and C such that: $\frac{1}{x^3-4x}=\frac{A}{x-2}+\frac{B}{x}+\frac{C}{x+2}$.

7) Determine the domain and range of $f(x) = \ln \sqrt{x^2 - x - 12}$. (Show thinking)

8) Use a computer graphing program (Winplot works nicely) to explore the graph of $y = 5\sin x - 3\cos x$. Plot this graph.

a) Attach a copy of your graph.

b) State the amplitude and period of your graph.

c) Write a rule for the function only using sine.

d) Use the identity $\sin \ \alpha - \beta$ to show that the curve $y = 5\sin x - 3\cos x$ can be written equivalently as $y = a\sin(x - p)$. Note: *a* and *p* should be to three decimal places.

9) A line is parallel to the vector $\langle -2, 5 \rangle$ contains the point (3, 4). State the equation of the line in standard form.

10) Solve over the complex numbers: $27z^3 + 8 = 0$

11) Dr. Condie decides to begin a workout program by running laps on the track. He has with him a die numbered 1, 2, 3, 4, 5 and 6. At the end of each lap he rolls the die, and if 2 or 4 comes up, he stops for the day, but if any other number appears, he continues running. So at the end of any lap he has a 2/3 chance of continuing and a 1/3 chance of stopping. The probability he runs exactly three laps is $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{27}$. Give two justifications for the equation

$$1 = \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^3 \cdot \frac{1}{3} + \ldots$$

12) Express 3.2565656… as a ratio of two integers with no common divisors using the techniques from the sequence unit.

13) Prove: $1 - \frac{1}{2}\sin(2x) = \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}$

14) Let $\vec{v} = \langle a, b \rangle$ and $\vec{u} = \langle c, d \rangle$. Give an argument that justifies the statement

$\vec{v} + \vec{u} = \vec{u} + \vec{v}$, that is, vector addition is commutative.

15) Let $\vec{v} = \langle a, b \rangle$ and $k > 0$. Show $k \cdot |\vec{v}| = |k \cdot \vec{v}|$.

16) Find the inverse of each function:

a) $f(x) = 2e^{3x-1}$ b) $h(x) = 2 + \sin(2 - 5x)$

Projectile Motion

From physics we know that an object projected upward at a velocity of v_0 ft/sec from an initial height of h_0 feet, has a height *h* feet after *t* seconds given by

$$h(t) = -16t^2 + v_0 t + h_0$$

If the velocity is in meters/sec, time in seconds, and height in meters we have

$$h(t) = -4.9t^2 + v_0 t + h_0$$

Also, the velocity of the object after *t* seconds is given by

$v(t) = -32t + v_0$ (English system) or $v(t) = -9.8t + v_0$ (metric system)

17) An object is projected upward from a height of 50 ft at a velocity of 132 ft/sec.

a) When does the object reach its maximum height?

b) State the maximum height.

c) Find the speed of the object when it hits the ground.