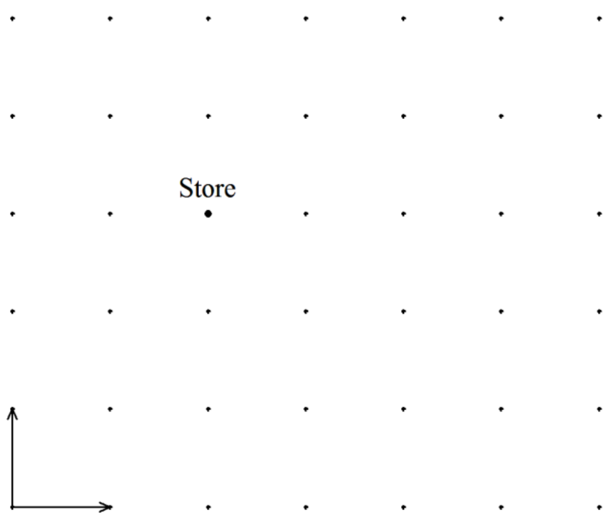
IMSA **MI-4** Prob. Set **#7** **DUE**: Thursday, March 24 Spring '11

1) Naomi is thinking of taking a random walk to get from her home to the store. In the grid below, Naomi's home is located in the lower left corner, (the origin) and the store is at (2, 3).



When Naomi starts, she flips a coin and if heads appears, she goes right, and ii tails, she goes up. She never goes beyond the store or goes backwards, so if she gets to (2, 0), she forgets the coin and walks only up.

a) How many different paths are possible?

b) List all the paths. Ex: RUURU is right-up-up-right-up.

c) Expand $R+U\ ^{5}$ (Use the Binomial Theorem if you know it.). There is something going on between this answer and the answer to part (a) which we will explore in future problem sets.

2) Simplify: $\sin(x-y)+\sin(x+y)$.

3) Solve the following for x where $-\pi \le x \le \pi$.

a) $2\sin(2x)+\cos(2x)=1$ b) $4\cos^2\left(3x-\frac{\pi}{6}\right)+3=0$

4) Simplify: $\frac{\sec^2 x}{\csc^2 x}-\frac{1}{\tan^2 x}$

5) In trigonometry there are product-to-sum formulas:

$$\sin\alpha\sin\beta=\frac{1}{2}\left[\cos\ \ \alpha-\beta\ \ -\cos\ \ \alpha+\beta\ \ \right]$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos\ \ \alpha-\beta\ \ +\cos\ \ \alpha+\beta\ \ \right]$$

$$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin\ \ \alpha+\beta\ \ +\sin\ \ \alpha-\beta\ \ \right]$$

a) Establish (prove) the second formula listed above.

b) Collaborate with a calculus student to do the following:

$$\int\cos(3x)\cos(x)\,dx=?$$

The calculus student will need to use the identity you proved. Have the calculus student do the work on you answer sheet and sign his/her work. Bonus: If you are in Mr. Kammrath's class and use one of his students, the calculus student will earn extra credit.

6) Evaluate: $\sum_{n=1}^{\infty}\frac{3^n-2^n}{7^{n+1}}$.

7) $\cos 1°+\cos 2°+\cos 3°+...+\cos 360°=$

8) A triangle has $m\angle A=17°$ and side $b=18$. Find the range of values for side a so that,

a. one triangle is possible.
b. two triangles are possible.

9) Willis Tower (formerly Sears Tower) is 1451 ft tall, with the Skydeck (observation deck) at 1353 feet. The advertising literature for the Skydeck claims a person can see for 40 – 50 miles. Imagine it is a clear day and you look out over Lake Michigan from the Skydeck. How many miles can he actually see? Assume the radius of the earth is 3960 miles. Caution, the distance is to be measured along the curvature of the earth.

10) $g(x)=2\cos\left(5x-\frac{\pi}{6}\right)+4$

a) Find $g^{-1}(x)$ and state its domain.

b) State the amplitude, period, phase shift, and vertical shift of g.

c) Rewrite g as a function of sine by applying the identity $\sin\left(\frac{\pi}{2}-x\right)=\cos x$ and simplifying.

11) Henry walks out his hall and proceeds 140 ft due North. He turns $25°$ left (west) and walks 300 ft. He then turns again $30°$ left and walks 175 ft. How far is he from his starting point?

12) Consider $f(x) = \sin^{-1} \ \sin x$.

a) Graph this function on your computer and attach with both axes labeled with multiples of $\frac{\pi}{2}$.

b) State the domain and range of this function.

c) State the coordinates of all the maximum points of *f*.

d) What is the period of *f* ?

e) $g(x) = \sin(\sin^{-1} x)$. Explain why this graph is not identical to the graph of *f*.

13) Find $\csc\left(\cot^{-1}\left(\sin\left(\cos^{-1}\left(\frac{-4}{5}\right)\right)\right)\right)$. (Show work.)

14) Solve: $\sin x + \cos x = 1 + \tan x,\ -2\pi \le x \le 2\pi$, by graphing.

a) Attach your graph. You graph should be labeled appropriately.

b) State your exact solutions.

15) Simplify, giving an exact value: $\tan\left(\sin^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{-4}{5}\right)\right)$ by applying the identity for $\tan \ \alpha + \beta$.

16) Consider the sequence of regular hexagons formed by connecting the midpoints of the sides of the previous regular hexagon. The dashed lines mean that the pattern continue indefinitely. If the length of the **side** of the largest hexagon is 12, find the ***sum*** of the **perimeters** of all of the hexagons formed in this way. Give an exact answer.