

- 1) Robin has a hobby of building doll house furniture. A model piano consumes 1.5 ml of paint and uses 25 cc of balsam wood. His good friend Madeline would like Robin to make one for her doll house which is scaled twice as large as Robin's. How much paint and how much wood will it take for him to produce a model piano that has twice the dimensions of his model?
- 2) Write an explicit formula for the sequence:

a. 
$$a_n = \begin{cases} 3 & \text{if } n=1 \\ a_{n-1} + 4 & \text{if } n > 1 \end{cases}$$

b. 
$$a_n = \begin{cases} 18 & \text{if } n=1 \\ \frac{2}{3}a_{n-1} & \text{if } n > 1 \end{cases}$$

3) Simplify: 
$$3 + \frac{2}{3 + \frac{2}{3 + \frac{2}{3 + \frac{2}{\dots}}}}$$

where ... indicates that the pattern continues indefinitely. Give exact answer and a six decimal approximation.

- 4) On your answer sheet, you see the graphs of the functions  $y = 2 \sin\left(2x - \frac{\pi}{4}\right)$  and  $y = 1$ .
- To find where these graphs intersect, you must find the value of  $\theta$  in Quadrant 1 such that  $2 \sin \theta = 1$ . What other quadrant on the unit circle must you also consider?
  - For the value of  $\theta$  in Quadrant 1, find the exact  $x$ -values of the points of intersection of the two graphs. Write your solutions in the form  $\alpha \pm k\beta$ . Be sure to use the variable  $k$ .
  - On the graph on your answer sheet, label the points of intersection with " $k = 0$ ," " $k = 1$ ," etc. for *all* values of  $k$  which correspond to solutions in the range shown on the graph.
  - For the value of  $\theta$  in the other quadrant, find the  $x$ -values of the points of intersection of the two graphs. Write your solutions in the form  $\alpha \pm m\beta$ . Be sure to use the variable  $m$ .
  - On the graph on your answer sheet, label the points of intersection with " $m = 0$ ," " $m = 1$ ," etc. for *all* values of  $m$  which correspond to solutions in the range shown on the graph.

The notation  $\binom{n}{r}$  means " $n$  choose  $r$ ", or the number of combinations of  $n$  objects taken  $r$  at a time, or the number of ways to group  $r$  objects from a set of  $n$  objects.

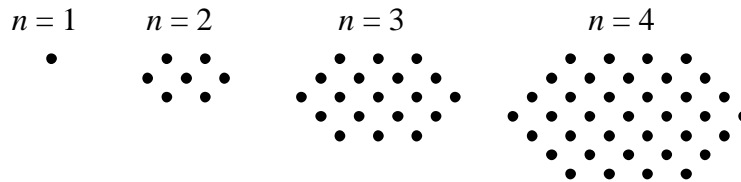
$$\text{It is equal to } \frac{n!}{r!(n-r)!} \text{ or } \frac{n(n-1)\cdots(n-r+1)}{r(r-1)\cdots 2 \cdot 1},$$

where  $n$  and  $r$  are non-negative integers and  $n \geq r$ .

5) NC a) Solve for  $n$ , if  $\binom{16}{n} = \binom{16}{n-2}$ .

NC b)  $\binom{n-2}{2} + \binom{n-3}{2} + \binom{n-4}{2} = 109$

6) In the following pattern, how many dots are there in the  $n$ th figure?



7) Find all values of  $n$ ,  $0 < n < 30$  such that

a)  $\sum_{k=0}^n \sin\left(\frac{k\pi}{6}\right) = 0$       b)  $\sum_{k=0}^n \cos\left(\frac{k\pi}{4}\right) = 0$

8) A tree grows vertically on a slope which has an incline of  $12^\circ$ . When the sun is at an elevation of  $50^\circ$ , the tree casts a shadow of 60 feet directly down the slope. What is the height of the tree?

9) Consider a sequence  $\{a_n\}$  such that  $a_1 + a_2 + \dots + a_k = k^2 + 3k$  for  $k = 1, 2, 3, \dots$

Show that  $\{a_n\}$  is not an arithmetic sequence and find a recursive formula for  $a_n$ .

10) Find a simplified formula, in factored form, for:  $\sum_{k=1}^{2n} k \cdot k+1$  using the formulas you learned in the sequences and series unit.

11) The first way most people learn to make Pascal's Triangle is by using addition of consecutive terms of one row to generate the next so

$$\begin{array}{cccccc} 1 & 4 & 6 & 4 & 1 & \text{yields} \\ 1 & 5 & 10 & 10 & 5 & 1 \end{array}$$

Show this is true in the general case, that is  $\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$  for  $0 < r < n$ .

12) If  $\cos \beta = x$  and  $0 < \beta < \frac{\pi}{2}$ , find exact values, in terms of  $x$ , for each of the following:

- a)  $\sin \beta$       b)  $\tan(-\beta)$       c)  $\sin(\pi - \beta)$       d)  $\sec(\beta - \pi)$

13) Find the sum of the first 100 terms of an arithmetic sequence  $\{a_n\}$  in which:

$$a_{12} = 42 \text{ and } d = -5$$

14) Consider the graph below which is the graph of  $y = a \tan(bx)$ .



a) State the domain of the function?

c) State the equation of this graph if it contains the point  $\left(\frac{\pi}{3}, -3\sqrt{3}\right)$ .

15) Consider the sequence: 
$$\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_n = 2a_{n-2} + 3a_{n-1} \end{cases}$$

a) State the first 8 terms of the sequence.

b) Let  $G_n = \frac{a_{n+1}}{a_n}$ . Find  $G_5$ .

c) Give a five decimal approximation of  $\lim_{n \rightarrow \infty} G_n$ . Hmm, this answer looks familiar.

16) Find the domain for each of the following:

a.  $f(x) = \frac{1}{e^{2x} - 5e^x + 4}$

b.  $h(x) = \sqrt{\log_4 x^2 + 9x + 19}$

17) Find real numbers  $A$ ,  $B$ , and  $C$  such that 
$$\frac{26x^2 + 53x + 44}{3x(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{3x}.$$