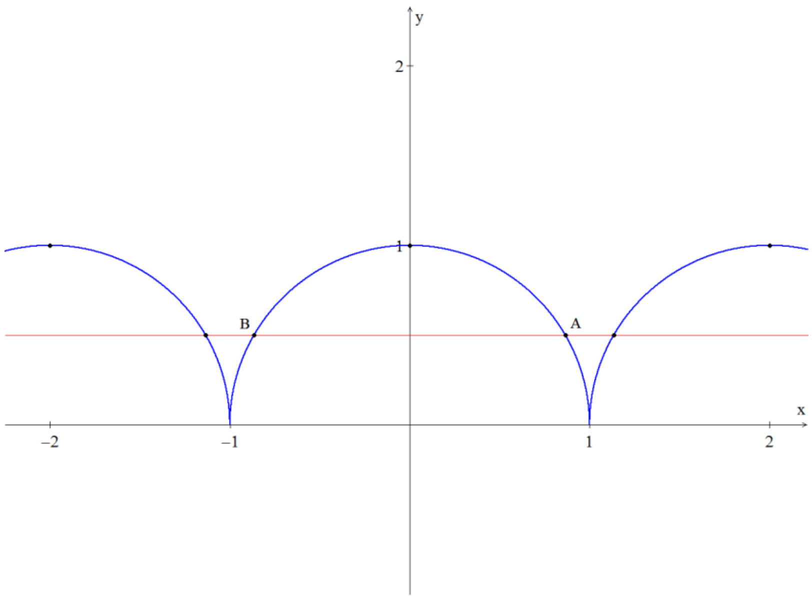
IMSA **MI-3** Prob. Set **#8** **DUE**: Thursday, Nov. 4, 2010 Fall '10

1) The graph below consists of semicircles of radius 1 that extend in both directions without end.



a) Find an expression that gives all values of *x* when $y = 1$. Several of the points are marked on the graph.

b) The line drawn on the graph is $y = \frac{1}{2}$. Find the coordinates of point A.

c) Find the coordinates of point B.

d) Find two general expressions that give all the points of intersection of the line with the circle. Your answers will look like: $x = \text{expression}$, $y = \text{expression}$ along with some explanation. Hint: Point A generates one set, point B another.

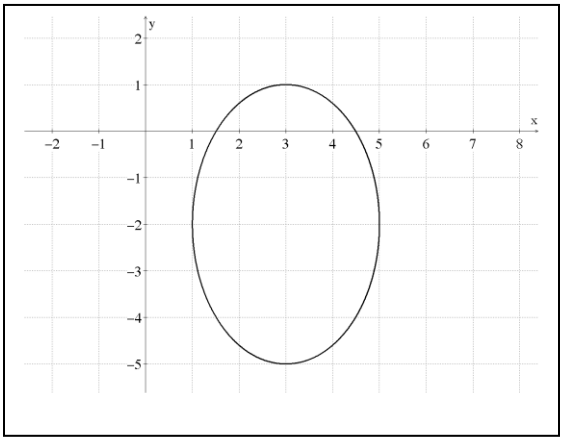
In a previous math course you learned that replacing *x* with $(x-h)$ and *y* with $(y-k)$ results in the translation $T: x \to (x+h),\ \ y \to (y+k)$, that is, on the graph any one point will have its *x*-coordinate increased or decreased by $|h|$ and its *y*-coordinate increased or decreased by $|k|$. In other words, the graph is shifted *h* units horizontally and *y* units vertically.

Example: Compare the graphs of $x^2 + y^2 = 9$ to $(x-3)^2 + (y+2)^2 = 9$

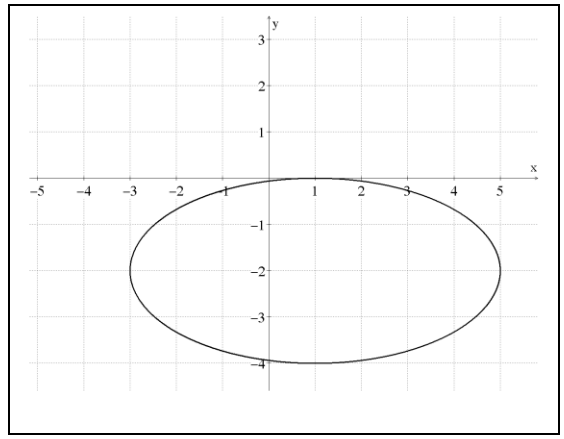
Solution: The first is a circle centered at (0, 0) with a radius 3. The second is the graph translated three to the right and 2 down, a circle of radius 3 centered at $(3, -2)$.

2) Use the above as well as your work on problem set #7 to state the equation of the ellipses graphed below. No need to show any work.

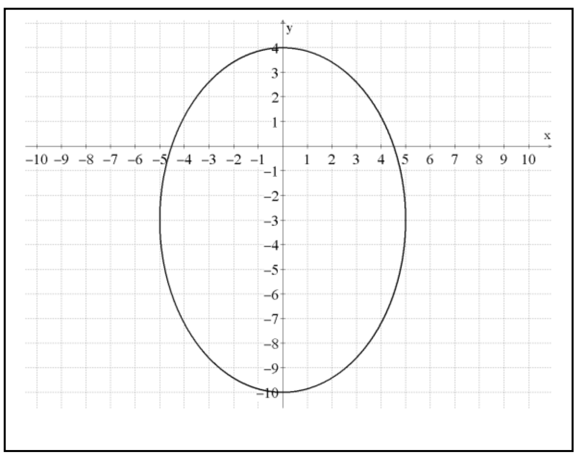
a)



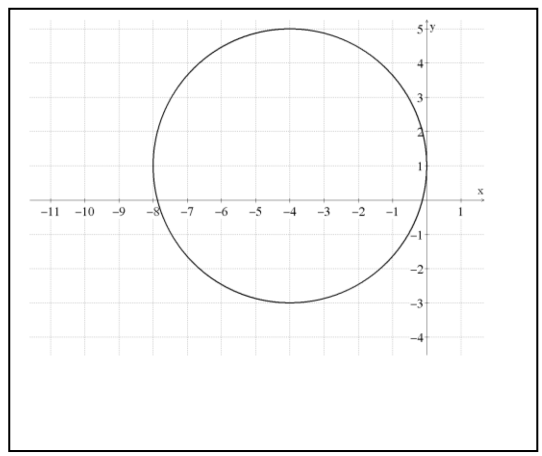
b)



c)

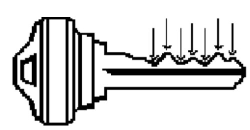


d)

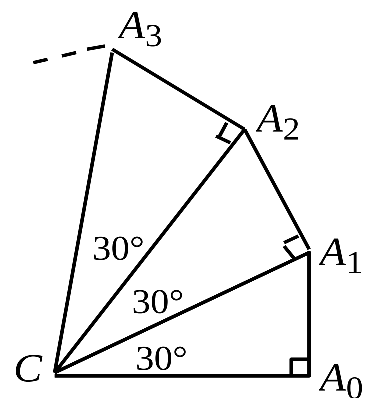


3) Find the angle of inclination (to the nearest tenth of a degree) of the line given by the equation $4y - 3x = 12$.

4) A certain key is cut for a lock with <u>seven</u> tumblers, each of which has <u>five</u> depths and no two consecutive tumblers can be the same depth. How many different keys can be made?



5) Solve for x: $\log_4(x^2) + 6\log_x(4) = 7$



6) Suppose the length of $\overline{A_0C}$ is 1 cm in the figure at the right.

   a. Find the length of $\overline{A_1C}$.

   b. If the pattern in the figure continues, the length of $\overline{A_5C}$

   c. If the pattern in the figure continues, the length of $\overline{A_nC}$.

7) **NC** Solve for x: $\begin{vmatrix} \log_5 x^2 & \log_6 36 \\ \log_4 64 & \log_2 5 \end{vmatrix} = 0$

8) Consider the function $f(x) = \dfrac{3x^3 - 20x^2 + 36x - 16}{3x^3 + 5x^2 - 26x + 8}$. Factor with calculator to find:

   a. all the zeros of $f(x)$.

   b. all the values of x for which $f(x)$ is not defined.

9) Find all asymptotes for each of the following functions. Show the work that leads to your answers.

   a. $f(x) = \dfrac{x^2 - x - 1}{3x^2 - 27}$

   b. $f(x) = \dfrac{2x - 5}{x^2 + 5x - 24}$

   c. $f(x) = \dfrac{4x^2 - 11x - 7}{4x - 3}$

10) Consider the function $f(x) = \dfrac{x^2 + 3x - 10}{x^2 + 7x + 10}$.

   a. Determine the domain and the range of $f(x)$.

   b. Find the vertical and horizontal asymptotes of $f(x)$.

   c. Sketch the graph of $y = f(x)$.

   d. Explain why $x = -5$ is not a vertical asymptote of $f(x)$.

11) Solve for z:

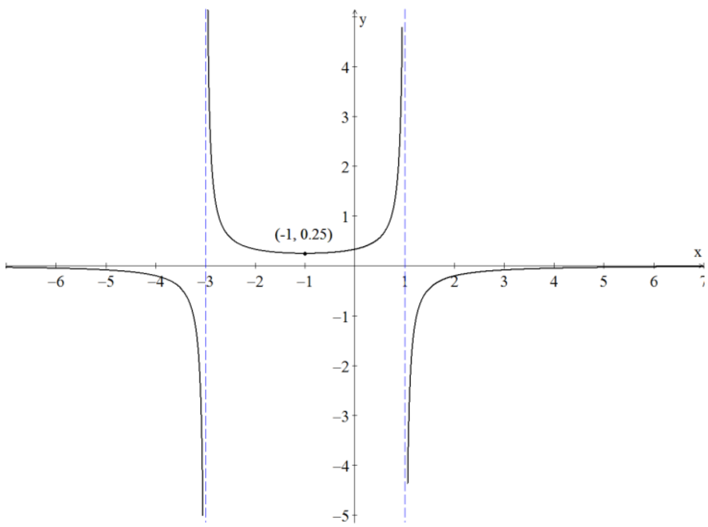
   a. $(3 + 4\boldsymbol{i})z + (-7 + 9\boldsymbol{i}) = 19 + 2\boldsymbol{i}$

   b. $z^2 = -11 - 60\boldsymbol{i}$

12) Find the inverse of each function.

a. $f(x) = \dfrac{2x-1}{3-5x}$ b. $g(x) = 2-3^x$

13) Give the equation of the reciprocal of the function graphed below. $x=-3$ and $x=1$ are vertical asymptotes. Leave your answer in standard form: $y = ax^2 + bx + c$.



14) When the *binomial* $(a+b)^5$ is expanded, the product a^3b^2 appears. After like terms are collected, what is the coefficient of a^3b^2? (Show the expansion. OK to use TI-89 to expand.)

15) If five pennies are tossed, what is the probability that exactly three of them will land showing tails?

16) A set has five elements.

a) How many subsets does it have? (Be careful to count the empty set, as it is a subset of every set.)

b) How many of those subsets contain the first three elements of the set?

c) If we randomly chose one subset of those you considered in part (a), what is the probability it contained the first three elements of the set?