

### Recursion On a Calculator

Recursion is a process of repeating the same operation over and over. Specifically in mathematics, we take an expression, usually a number, apply a rule to get another expression or number, and apply the rule to that result. For example consider the sequence of numbers:

$$4, 7, 10, 13, \dots$$

The recursive rule is 'add three'. We write this in mathematics as

$$a_n = a_{n-1} + 3$$

On the calculator you can generate these number by doing the following:

1. Type 4 and enter. This stores the number 4 in memory labeled 'answer'.
2. Press 'ans' + 3. This takes the previous result and adds 3. (Note: on most calculators you could just enter+3.)
3. Now 7 appears on the screen. Press enter and the calculator takes that answer plus 3.
4. Continue as desired.

- 1) Generate the terms of the sequence  $(a + b), (a + b)^2, (a + b)^3, \dots, (a + b)^6$  using the techniques described above and the 'expand' key. Start by typing  $(a + b)$  pressing enter, then times  $(a + b)$ . Your answer should be written in expanded form.
- 2) Mr. Jones has hired his neighbor, Robin to look after his salt water fish tank while he travels the world. The tank is a 50 gallon tank. When first set up, Mr. Jones put 8 lb of salt in the 50 gal. to create the correct salinity. It is a good practice in keeping fish to replace 10% of the water each week. Each week, one gallon of water evaporates, but no salt is lost, so the tank still contains 8 lb of salt, only in 49 gallons.
 

After one week, Robin proceeds to add enough fresh water to replace the evaporated water, then removes 5 gal. of tank water and adds 5 gal. of fresh water. Robin is now creating a problem for the fish in that the salinity of the water has been reduced.

  - a) Using your calculator and its recursive ability, write a sequence of terms that gives the remaining pounds of salt in the tank each week. Your first term of this sequence should be 8. Give 5 terms of this sequence.
  - b) Below 4 lb. of salt in 50 gal. of water, the fish in the tank become very stressed. When does this occur?

Robin's friend, Naomi realizes that removing the water also removes the salt. She suggests he add a bit of salt with each water change. They decide on this plan. Remove 5 gal of water, fill the tank back up, and add  $\frac{1}{2}$  lb. of salt. Unfortunately, they did not take into account evaporation, so when they filled the tank up, they added 6 gal. of water and  $\frac{1}{2}$  lb of salt, they were still changing the salinity of the tank.

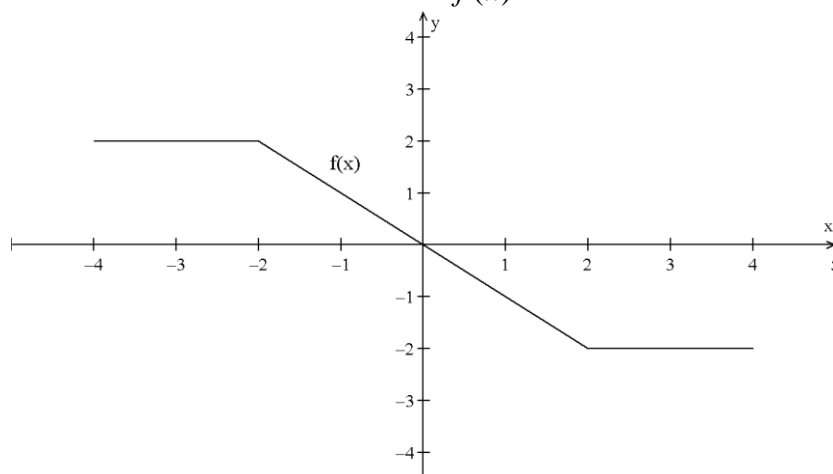
- c) Determine the amount of salt remaining in the tank at the end of each exchange with this new plan. At time 0, you should have 8 lb., and at time 1, what is left after the first exchange. Use Excel to do this problem, generating a table for 20 weeks. On your answer sheet write the exact recursive formula you entered in Excel.
- d) Plot you data in part (c) using the scatter plot feature of Excel. Note: You are modeling a discrete function, so it would not be appropriate to connect the points. Attach a printed version of your Excel document that includes your table and the graph, correctly labeled.
- e) When will the amount of salt be 4.8 lb? Explain.

3) Function  $f$  is graphed below.

a) Graph the reciprocal of the function  $f(x)$ , that is graph  $y = \frac{1}{f(x)}$ .

b) State the domain and range of  $f(x)$ . Use interval notation.

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4) 
$$\text{NC} \sum_{n=1}^{2010} i^n =$$

5) NC  $\sum_{n=1}^{2010} \left(\frac{1}{i}\right)^n =$

6) Use long or synthetic division to divide  $x^3 - 4x^2 + 3x - 2$  by  $x - 5$ . Make sure you show all your work and clearly state the quotient and remainder.

7) Find the domain of the function  $f(x) = \sqrt{\frac{x^2 - 4x - 21}{x^2 - 4x - 5}}$ .

8) NC Simplify:

a.  $\frac{6! - 5!}{5!}$

b.  $\frac{11! - 10!}{10!}$

c.  $\frac{n! - (n-1)!}{(n-1)!}$

In the polynomial unit, you learned long division of polynomials. A proper fraction in arithmetic is one that is reduced and the numerator is smaller than the denominator. Similarly, a proper fraction in algebra is one with no common factors and the degree of the numerator is less than the degree of the denominator. The TI-89 has a 'propFrac' command on the algebra menu that will give you the result of a division problem.

Example: Using propFrac, with the fraction  $\frac{x^3 - 4x^2 + 3x - 2}{x - 5}$  yields

$x^2 + x + 8 + \frac{38}{x - 5}$ , which is the quotient and remainder of dividing  $x^3 - 4x^2 + 3x - 2$  by  $x - 5$ . (This answer should look a bit familiar.)

9) Use your calculator to find the quotient and remainder of the following. No work need be shown.

a)  $x^6 + 3x^3 - 5x^2 + 3 \div x^2 - 5x + 7$

b)  $x^{10} - 8 \div (x - 2)$  [Interesting answer!]

10) Find the remainder when the given function in the first column is divided by the function in the second. Use your calculator for this problem.

Function $f(x)$	Divisor
a) $x^3 - 3x^2 + 8x + 5$	$x - 1$
b) $2x^2 - 10x + 3$	$x - 3$

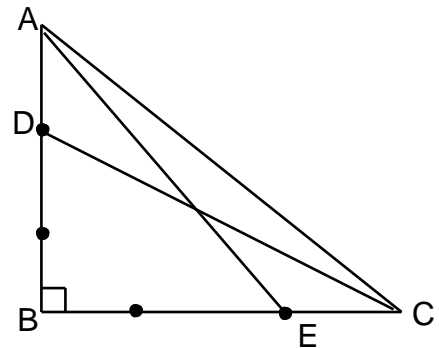
c) $-3x^4 + 5x^2 + 8x + 10$	$x + 2$
d) $x^3 - 3x^2 - 7x - 15$	$x - 5$

11) For each of the polynomial functions in problem 10, find the following values using your calculator.

- $f(1)$  (Use the rule in part (10a).)
- $f(3)$  (Use the rule in 10b.)
- $f(-2)$  (Use the rule in 10c.)
- $f(5)$  (Use the rule in 10d.)

12) Make a conjecture concerning your answers in problems (10) and (11) and justify your conclusion. Historical note: This problem is why synthetic division was also called synthetic substitution.

- 13) The legs of right triangle ABC are trisected as shown in the figure at the right. If  $AE = 8$  and  $DC = \sqrt{53}$ , find AC.



- 14) Complete the square to find the center and the radius of the circle given by the equation:

$$3x^2 + 3y^2 - 12x + 42y + 15 = 0$$

- 15) Use a computer graphing program to graph the following. If you use Winplot, you will want to graph in implicit mode so you do not have to solve for  $y$ . Print the following on separate paper, well labeled, and attach to your answer sheets.

a) Graph the ellipses on the same grid:

$$\frac{x^2}{36} + \frac{y^2}{25} = 1 \qquad \frac{x^2}{25} + \frac{y^2}{36} = 1$$

b) On another grid graph:

$$25x^2 - 100x + 16y^2 - 160y + 100 = 0$$

16) Complete the square on question 15b to put it into the standard form for an ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

17) One can look at an ellipse as a transformation of a circle, so a circle centered at the origin has the general form  $x^2 + y^2 = r^2$ , an ellipse centered at the origin has the form  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

Note if  $a = b$  the equation of the ellipse becomes the equation of the circle. In this problem you will explore ellipses using sliders in your graphing software.

Your task is to write a set of guidelines explaining how to graph an ellipse centered at the origin, given its equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and explain the statement above 'One can look at an ellipse as a transformation of a circle'. Be thorough in your explanation (full sentences please). Your audience should be a fellow MI 3 students that did not do this exploration. If you need more space, attach another page.

Winplot Instructions for creating a slider.

- Under the Anim menu, choose parameters
- Set A to 1 and B to 1 by typing 1 in box and enter
- Under equation choose implicit
- Type the equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- When you press enter you should see a circle of radius 1
- Return to Animate and move the slider for  $a$  or  $b$  to change the values in the equation