General Directions for the Problem Sets::

The purpose of these problem sets is threefold:

a. To review, enhance and make connections with your past knowledge

b. To work with current concepts and skills being discussed in class in both familiar and new contexts.

c. To preview ideas and techniques that will become important in the near future. In addition, some ideas will be introduced/ explained in the problem sets. Usually the first time such an explanation is presented it will appear in a box. You should be sure to pay special attention to such boxed directions as you will be expected to be able to **use that information from then on**.

In addition, you will regularly see material that has been studied earlier in the course or in previous MI courses. If there are any of these problems that you have never seen or are rusty on, it is your responsibility to seek outside help to solidify that skill.

Detailed - Correct - Neat & Legible Solutions are to be written on the answer sheets provided and turned in by the due date. **You are permitted to obtain help from books, your teacher, other students, or the math department I.P.A. in order to clarify anything that you don't understand, but the work must be your own.** Be sure to review anything that you needed help with, in order to be sure that you can do it by yourself. **Collaborative work that supports learning is encouraged and assumed in Mathematical Investigations, however, under no circumstances should you loan your problem set to another student – it supports cheating and blatant copying and you will be held responsible for such action.**

You are ultimately held accountable for being able to perform any tasks related to the problems.

The Problem Sets are to be completed along with your regular math class homework, which will be shorter and with a different emphasis than the Problem Sets. Budget your time to allow for you to get help when needed, **before the due date**. It does you no good to rush through these problems the night before they are due, because some of the concepts will be needed during the class lessons that meet before the due date.

You are expected to maintain a 3-ring loose-leaf notebook containing the following:

1. Course information
2. Formula List
3. Vocabulary List
4. "Things that are not clear to me."
5. Class explorations & worked out solutions
6. Other "handouts"
7. Problem sets & your answer sheets
8. Other important stuff

Tabs should be used to separate major sections.

All answers should be exact whenever possible.
Approximated answers should be rounded to four significant digits unless otherwise stated in the problem.

1. If the real roots of $y = f(x)$ are $\{-4, 3, 8\}$ (i.e. $f(-4) = 0$, $f(3) = 0$ and $f(8) = 0$), find the real roots of: (state exactly)

   a. $y = f(x-5)$ b. $y = f(2x)$

   c. $y = f(x^2)$ d. $y = f(x^3 - 2)$

2. Given $x^2 - 15x + 36 = 0$,
   a. What is the sum of the roots?
   b. What is the product of the roots?
   c. If a and b are the two roots, what is $a^2 + b^2$?

   [Note: You do not need to solve the equation in order to answer these questions.]
   [Hint: If $x^2 + 3x - 18 = 0$, then $(x - a)(x - b) = 0$ where a and b are the roots of the equation.]

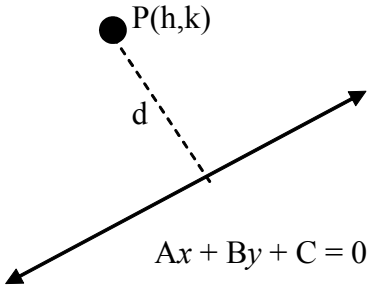
3. If 0.6% of a radioactive substance decays every year,
   a. what percent (to the nearest tenth of a percent) will be left after 35 years?
   b. how many **full** years will it take until less than half of the substance remains?

4. Find the set of all integers, n, for which the fraction $\dfrac{16}{18-n}$ is a positive integer.

5. Solve for x: a) $5^{2x-3} = 25^{3x+1}$ b) $16^{4x+3} = 8^{3x-5}$ c) $5(2^{3x}) + 3(8^x) = \dfrac{2^x}{64}$

6. The distance between a point, P(h,k), and the line, Ax + By + C = 0, can be found by evaluating the expression:

$$d = \frac{|Ah + Bk + C|}{\sqrt{A^2 + B^2}}$$



   a. Find the distance from the point (4, 2) to the line $3x + 10y = -8$.

   b. Find the distance between the lines $3x + 4y + 7 = 0$ and $2x + 4y - 1 = 0$.

7. A *perfect square trinomial* is any trinomial of the form $u^2+2uv+v^2$. For example $4x^2-12x+9$ is a perfect square trinomial because it equals $(2x-3)^2$. Here $u=2x$ and $v=-3$. Similarly $x^2+14xy+49y^2$ is a perfect square trinomial because it equals $(x+7y)^2$. Here u = x and v = 7y.

On each of the following, fill in the boxes and the blanks so that each expression is a perfect square trinomial:

a. $x^2+6bx+\square=(x+\_\_\_\_\_)^2$

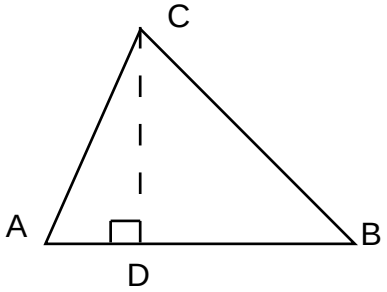
b. $x^2-12xy+\square=(x-\_\_\_\_\_)^2$

c. $36x^4-60x^2y+\square=(\_\_\_\_-\_\_\_\_)^2$

8. **NC** (No calculator) If $f(x)=\dfrac{3x+15}{x-8}$, find $f^{-1}(x)$.

9. If $CD=8$ and $AB=3x-6$,

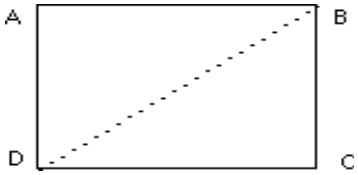
a. find A(x) = the area of ΔABC
b. find the domain of A(x) algebraically.



10. **NC** Simplify: a. $\left(\dfrac{5a^3b^2}{2c^2d^4}\right)^2 \div \left(\dfrac{ab^3}{c^4d^4}\right)^3$ b. $\dfrac{3^{n+4}-3\ 3^{n-1}}{27\ 3^{n+1}}$

11. If the area of rectangle ABCD is 62 and its perimeter is 30, find the length of diagonal BD.

Note: You do not need to find the length of the sides.



12. a. If $b^2 > b$, what can you conclude about the value of b? Justify your answer.
b. If $0 < b < 1$, arrange b^3, b^{-3}, b, and $b^{1/3}$ in order from smallest to largest. Justify your answer.

13. Make a sketch each function and give its domain and range. Your sketch should clearly show significant points. Give your answer in interval notation.

a) $f(x)=4-x^2$ b) $g(x)=\sqrt{36-x^2}$

14). Below is a table of the population of the United States from 1790 (when the first census was conducted) through 2000.

1790 - 3,929,214
1800 - 5,308,483
1810 - 7,239,881
1820 - 9,638,453
1830 - 12,866,020
1840 - 17,069,453
1850 - 23,191,876
1860 - 31,443,321
1870 - 38,558,371
1880 - 50,189,209
1890 - 62,979,766
1900 - 76,212,168
1910 - 92,228,496
1920 - 106,021,537
1930 - 123,202,624
1940 - 132,164,569
1950 - 151,325,798
1960 - 179,323,175
1970 - 203,302,031
1980 - 226,542,199
1990 - 248,709,873
2000 - 281,421,906

a. Consider the first seven census's, from 1790 through 1850. Let *t* represent years from 1790, ie, 1790 is 0, 1800 is 10 and so on. Using the technique you used on page 1.3 of your MI 3 notebook, find an equation that models the population of the United States as a function of time *t* for the period **1790 – 1840**. Show your steps including a table, logarithmic equation, and final equation. Use three decimal places throughout this problem. Note: You will learn how to do this using other technology, but for now, use your calculator and enter your results on your answer sheet by hand.

b. Use your model to predict the population of the U.S. in 2000. Comment on the appropriateness of the model. If your prediction is not close to the actual population, conjecture why it was different.