1) Suppose that IMSA were to give three awards to math teachers – one to the teacher whose students average the greatest number of hours of homework each week, one to the teacher who had the highest number of compliments on the student surveys, and one to the teacher who wrote the greatest number of problem set questions. Assuming a teacher may receive at most one award, in how many different ways can these awards be given if there are:

   a) 12 teachers on the Math Team   b) 15 teachers on the Math Team

2) $A = \{a, b, c, d, e, f\}$.

   a) How many two element subsets does *A* have? List them.

   b) How many four element subsets does *A* have? List them.

   c) Explain why the number of subsets in part a) and part b) are the same.

   d) A set has 8 elements. How many two element subsets does it have?

Parametric Equations

Let $x = f(t)$ and $y = g(t)$, where *f* and *g* are two functions whose common domain is some interval I. The equations

$$x = f(t) \qquad y = g(t)$$

Are called **parametric equations with parameter *t***.

Example: $\begin{cases} x = 3t - 2 \\ y = 5t + 4 \end{cases}, \; 0 \le t \le 3$

As we let *t* vary between 0 and 3 we get sets of points. Some values are given below.

t	*x*	y
0	-2	4
1	1	9
2	4	14
3	7	19

3) Consider the parametric equations $\begin{Bmatrix} x = t + 3 \\ y = 2t - 5 \end{Bmatrix}$ for $-2 \le t \le 5$.

a) Find the range values of *x*
b) Find the range values of *y*
c) As *t* changes by 1, at what rate does *x* change?
d) As *t* changes by 1,what rate does *y* change?
e) What is the *y* intercept of this graph?
f) Graph the set of points defined by these equations.

Recall from Geometry that if the three angles in one triangle are congruent to the three angles in another triangle, then the two triangles are similar. If the triangles are similar, then the corresponding sides are proportional. We also know that if two angles of one triangle are congruent to two angles of another triangle, the two triangles will be similar. Finally, recall that if an acute angle of one right triangle is congruent to an acute angle of another right triangle, then the two triangles are similar. Therefore, if a right triangle has a specific acute angle, then the various ratios of the sides will always have a specific value.

From this information, mathematicians have developed an important area of math called trigonometry. Three of the common ratios of the sides are defined as follows:

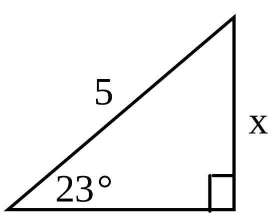
SINE: $\sin(a) = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$

COSINE: $\cos(a) = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$

TANGENT: $\tan(a) = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$

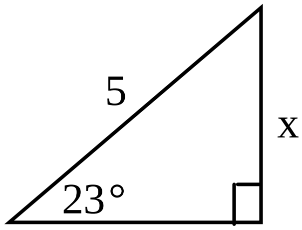
The values of these ratios are "built in" your calculator. Be sure that you check/set the mode to degrees whenever the problem indicates the angles are measured in degrees.

Example 1:



$\sin(23°) = \dfrac{x}{5}$, $x = 5 \cdot \sin(23°)$, $x = 1.9537$

Example 2:



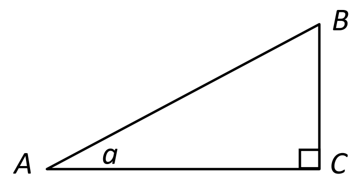
$\tan(35°) = \frac{x}{17.9}$, $x = 17.9 \cdot \tan(35°)$, $x = 12.5337$

4) *a*. $AB = 57.9$ and $\angle A = 27.4°$

Find BC and AC, each answer to the nearest tenth.

b. $AC = 36.8$ and $BC = 21.5$

Find $\angle A$ to the nearest tenth of a degree.

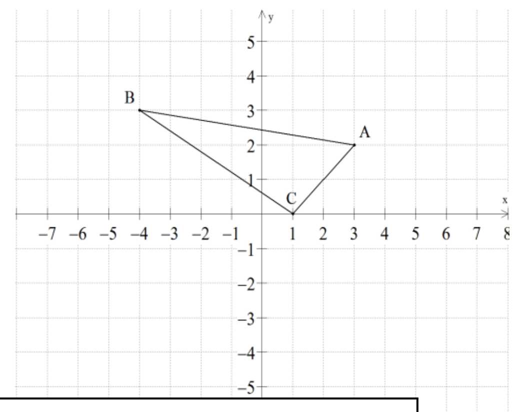


5) Given the figure at the right,

a) Reflect ΔABC over the line $y = -x$. Find new vertices of $\Delta A'B'C'$ and graph.

b) Rotate ΔABC 270° ciubterclockwise around the origin. Find new vertices of $\Delta A''B''C''$ and graph.

c) Find perimeter of ΔABC. State exactly.



Distance from a Point to a Line

The distance from a given point $P(x_1, y_1)$ to a given line, $ax + by + c = 0$, can be found using the formula: $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$.

6) Find the distance from point A to line containing BC in problem 5.

7) Solve for x by using the zero product property. Note, you can use your solutions to (a) to quickly find the solutions to the other problems.

a) $x^2 - 25x + 100 = 0$

b) $x^4 - 25x^2 + 100 = 0$

c) $\sqrt{x}^{\,2} - 25\sqrt{x} + 100 = 0$

d) $(x+1)^2 - 25(x+1) + 100 = 0$

8) a) Graph: $\left\{\begin{matrix} 3x - 2y = 6 \\ \textit{and} \\ 3x - 2y = -12 \end{matrix}\right\}$ on the same axes. [Label each graph.]

b) Graph $-12 \geq 3x - 2y \geq 6$.

9) The following data consist of the ages of actors and actresses who have won the Academy Awards from 1981 to 2010.

Actors:

37	76	39	53	45	36	62	43	51	32
42	54	52	37	38	32	45	60	46	40
36	47	29	43	37	38	45	50	48	60

Actresses

31	74	33	49	38	61	21	41	26	80
42	29	33	36	45	49	39	34	26	25
33	35	35	28	30	29	61	32	33	45

Use the *Data-Matrix Editor* on your calculator (or Excel) to compute following for each group:

a) Mean b) Five number summery c) Interquartile range d) Range

10) Make parallel box plots of the sets of data in problem 9.

11) Make two observations you note in comparing the ages of actors and actresses.

Completing the Square

When the zero property is not helpful in solving a quadratic equation, a method called *completing the square* can be used to create a perfect square trinomial on one side of the equation.

Example 1: assume we need to solve: $x^2 + 12x + 2 = 0$.

We wish to convert the left side to a perfect square; that is, we want it to be of the form:

$$a^2 + 2ab + b^2 = \ a + b \ ^2 .$$

Add -2 to both sides of the equation: $x^2 + 12x = -2$

In order to complete the square, add 36: $x^2 + 12x + 36 = -2 + 36$

Factor the left side: $x + 6 \ ^2 = 34$

Solve: $x + 6 = \pm\sqrt{34}$

and $x = -6 + \pm\sqrt{34}$

Example 2: solve $2x^2 - 6x + 1 = 0$. ***Note:*** **First divide by 2:**

$$\left(\frac{-3}{2}\right)^2 = \frac{9}{4}$$

$$x^2 - 3x + \frac{1}{2} = 0$$

$$x^2 - 3x = -\frac{1}{2}$$

$$x^2 - 3x + \frac{9}{4} = -\frac{1}{2} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{7}{4}$$

$$\left(x - \frac{3}{2}\right) = \pm\frac{\sqrt{7}}{2}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{7}}{2} = \frac{3 \pm \sqrt{7}}{2}$$

12) Solve for x by completing the square. (Show all work.)

a) $x^2 + 12x - 7 = 0$ b) $x^2 - 16x - 7 = 0$ c) $2x^2 - 16x - 9 = 0$