BC 3   More on Taylor   Name:_______________________

(1) Write out series for the following:

$\cos(x^3)=$

$\frac{\sin(2x)}{x} =$

$xe^x =$

$\frac{3x}{1-2x} =$

$\frac{x^2}{e^{2x+1}} =$

(2) Write out Taylor's formula with remainder for $f(x) = \sin x$, with $a = 3\pi/4$, and $n = 3$.

(3) If sin$x$ is to be approximated by $x - x^3/3!$, with $|x| < .4$, find an upper bound for the error.

(4) If terms through $n = 8$ are used to approximate e^2, find the error.

(5) What values of x can be used if $\cos(2x)$ is to be approximated by two non-zero terms with an error less than .005?

(6) Find the value of each series by recognizing the function and the point at which it is evaluated.

$$1 - \frac{(.2)^2}{2!} + \frac{(.2)^4}{4!} - \dots$$

$$1 + \frac{1}{2} + \frac{(1/2)^2}{2!} + \frac{(1/2)^3}{3!} + \dots$$

$$1 - .4 + \frac{(.4)^2}{2} - \frac{(.4)^3}{6} + \dots$$

(7) Use series to approximate $\int_0^{0.9} \cos(x^2)\, dx$ with an error less than .005.

(8) Use series to find $\lim_{x \to 0} \frac{\ln(1+x)}{\sin(2x)}$. Then use l'Hôpital's Rule to confirm your answer.

(9) If $f(x) = \sum_{n=0}^{\infty} (x/2)^n$, find the interval of convergence for f, for f', and for F.

(10) Let $f(2) = 1$, $f'(2) = -2$, $f''(2) = 4$, and $|f^{(n)}(x)| < 6$ for all x.
Find the Taylor series for f at $a = 2$, showing 3 terms.

If these three terms are used to approximate $f(2.3)$, find an upper bound for the error.