BC 3 Series 9 Name:_____

New from Old

Complete the right side of the equations below with the appropriate series and the interval of convergence if requested.

(1) $\sin x =$

Differentiate both sides.

Does this "work"?

Differentiate both sides again. Does this still "work"?

(2) $e^x =$

Integrate both sides.

Does this "work"? What do you <u>always</u> remember to include when integrating?

(3) Find each series. Show four terms. <u>Interval of convergence</u>

$\frac{1}{1-x} =$

$\frac{1}{1+x} =$

$\ln(1+x) =$

Note the endpoints, particularly on the integral.

(4) Find each series. Show four terms. Interval of convergence

$\frac{1}{1-x} =$

$\frac{1}{1+x^2} =$

$\tan^{-1}x =$

Note that this is the second method we've seen for finding the Maclaurin series for $\tan^{-1}x$. Any preference?

Theorem: (Simplified) Taylor series may be differentiated or integrated term by term and will represent the appropriate function on the same open interval.

Note: Convergence or divergence may change at each endpoint.

(5) Find, using simple multiplication.

$x^2e^x =$

$4x\cos(2x) =$

(6) Multiplication of series. Find the series for $\frac{e^x}{1-x}$ by multiplying enough terms of each of the two series together. (Be careful. Think ahead.)

(7) Let $f(x) = \dfrac{5x - 1}{x^2 - x - 2}$. Finding the derivatives of this to create the series directly would be somewhat unpleasant. Try this approach:

(a) Use partial fractions to decompose f.

(b) Write the series for each of the two partial fractions. (One may require a bit of sneakiness.)

(c) Add and simplify.

(8) Let's try that last function by an additional method: Long division. Find three terms.

$$-2 - x + x^2 \overline{)-1 + 5x}$$