Back to $f(x) = \cos x$ ...

We found a polynomial that can be extended to give

$$P(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

Let $P_n(x)$ refer to the polynomial approximating the function f which has "order of contact n" at a point $x = a$. That is to say, $P_n^{(k)}(a) = f^{(k)}(a)$ for $k = 0, 1, 2, 3, \ldots, n$, and here we have $a = 0$.

For example, with $f(x) = \cos x$, we have

$$P_4(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \quad \text{and} \quad P_7(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

Note that "n" can cause problems. Does it mean the nth term? the order of contact? the value of n in the summation? Just be careful!

(1) If we choose $P_4(x)$, giving us 3 (non-zero) terms, evaluate $f(x)$ and $P_4(x)$ for the following values of $x = a$.

	$f(a)$	$g(a) = P_4(a)$
$a = 2$		
$a = 1$		
$a = 0.5$		

For what values of x does P_4 seem to be a good approximation of f?

(2) Again using 3 terms, let $x = 1$. How big will the error be? This time, do this two ways and note the difference.

(a) Find $|\cos 1 - P_4(1)|$. (Simply evaluate $\cos 1$ on your calculator.)

(b) Use the alternating series error approximation.

(3) Still using 3 terms of P, what are the possibilities for x if the error is to be less than 0.00005?

Set the window on the calculator so that $-7 \le x \le 7$ and $-2 \le y \le 2$.

(4) Plot $P_4(x)$ and $f(x)$. Sketch the graphs below.

For what values of x does P_4 seem to be a good approximation of f ?

(5) Plot $P_6(x)$ and $f(x)$. Sketch the graphs below.

For what values of x does P_6 seem to be a good approximation of f ?

(6) Plot $P_8(x)$ and $f(x)$. Sketch the graphs below.

For what values of x does P_8 seem to be a good approximation of f ?

(7) Plot $P_{10}(x)$ and $f(x)$. Sketch the graphs below.

For what values of x does P_{10} seem to be a good approximation of f ?

(8) Plot $P_{14}(x)$ and $f(x)$. Sketch the graphs below. (Note: Skipped a graph!)

For what values of x does P_{14} seem to be a good approximation of f ?

(9) Based on these graphs, can you make any guesses about the values of x for which the infinite series P converges?

Do you think that there is a value of n such that $P_n(x)$ will have an error less than 0.0001 for all x in the interval $-1000 \le x \le 1000$? Why or why not?

(10) Determine the values of x for which the infinite series converges by using an appropriate test.

(11) Compare this result to the intervals found for $y = \ln(1 + x)$ and $y = \tan^{-1} x$.

Can you think of any possible explanations for this distinction?

More on error analysis...

(12) Using $P_{14}(x)$ and $x = 6$, what will the error be?

(13) Again using $P_{14}(x)$, what are the possibilities for x if the error is to be less than 0.0005?

(14) If $x = 4$, how many terms are necessary to be sure that the error is less than .01?