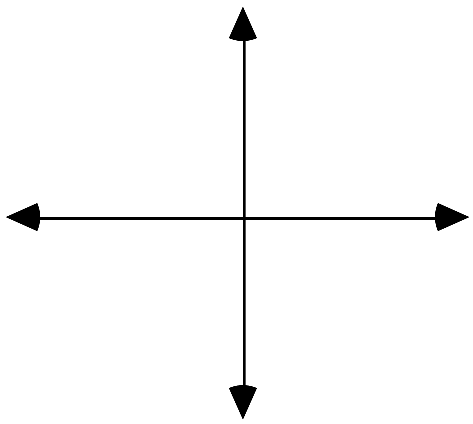
BC 3   Series 2   Name:________________________________

(1) Let $f(x) = \tan^{-1}x$ and let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. Set $f(0) = g(0)$, and set the first four derivatives evaluated at $x = 0$ equal to each other in order to find the values of the a_i and the polynomial $g(x)$. (In other words, set $f^{(k)}(0) = g^{(k)}(0)$ for $k = 0, 1, 2, 3, 4$.) Be careful with the derivatives of f. They aren't real friendly.

(2) Find both $f(a)$ and $g(a)$ for the following values of a. What do you notice?

	$f(a)$	$g(a)$
$a = 2$		
$a = 1$		
$a = 1/2$		
$a = 0.1$		

(3) Use your calculator to graph both functions *f* and *g*. Try different windows to get a good view and sketch the graph below. On what interval does *g* seem to be useful as an approximation of *f*?



(4) Continue the pattern, extending *g* to form an infinite series that will approximate the function *f*. (Yes, you only have two terms, so the pattern isn't very clear. Hints: The terms do alternate, and the pattern is nice and simple.) Use an appropriate test to determine the open interval for which this series converges.

Check the endpoints of your interval to determine whether the series will also converge at each of those values. State the entire interval on which the series converges.

(5) If we use only two terms of g and we use $x = 0.75$, find an upper bound for the error. Use this to find an interval for S.

(6) Again using these two terms, what are the possible values of x if the error is to be less than 0.005?

(7) If $|x| < 0.5$, how many terms of the series do you have to use in order to get an error which is less than 0.001 ?