BC 3 Series 1 Name:______________________________

(1) Let $f(x) = \ln(1 + x)$ and let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3$. Set $f(0) = g(0)$, and set the first three derivatives evaluated at $x = 0$ equal to each other in order to find the values of the $a_i$ and the polynomial $g(x)$. (In other words, set $f^{(k)}(0) = g^{(k)}(0)$ for $k = 0, 1, 2, 3$.)

(2) Find both $f(a)$ and $g(a)$ for each of the following values of $a$. Are they very close to each other? When are the values closer?

| | $f(a)$ | $g(a)$ |
|---|---|---|
| $a = 1$ | | |
| $a = 0.5$ | | |
| $a = 0.1$ | | |

(3) Use your calculator to graph both functions $f$ and $g$. Try different windows to get a good view and copy the graph below. On what interval does $g$ seem to be useful as an approximation of $f$?

(4) Now sketch a graph of $R(x) = | f(x) - g(x) |$. This represents the remainder, or the distance between the function $f$ and its approximation $g$. Describe the graph and what this says about $g$ as an approximation.

(5) Extend $g$ by continuing the pattern of the coefficients of $g$ to form an infinite series that will approximate the function $f$. Use an appropriate test to determine the open interval for which this series converges.

Check the endpoints of your interval to determine whether the series will also converge at those values.

(6) If we use only the three terms of our polynomial $g$ — the first three terms of our infinite series — and we use $x = 0.75$, find an upper bound for the error. Find $S_3$ and use this to write an inequality for $S$, the infinite sum.

(7) Again using these three terms and assuming $x > 0$, what are the possibilities for $x$ if the error is to be less than 0.005?

(8) If $0 < x < 0.5$, how many terms of the series do you have to use in order to get an error which is less than 0.001 ?