BC 1 Limits 4 Name:_____

Defn: A function f is continuous at a point $x = a$ if

(1) $f(a)$ exists,

(2) $\lim_{x \to a} f(x)$ exists,

and (3) $\lim_{x \to a} f(x) = f(a)$.

In other words, the function exists, the limit exists, and the functional value equals the limit. If any of these conditions does not hold, we say the function is discontinuous at $x = a$.

(1) Determine the points of discontinuity (x-values) for each of the following functions.

(a)

(b) $f(x) = \dfrac{x}{(x-1)(x+2)}$

(c) $g(x) = \dfrac{x}{x(x+3)}$

-2 1 3

(2) (a) Show that f is discontinuous at $x = 3$. Justify by checking each of the three conditions above.

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, & x \neq 3 \\ 4, & x = 3 \end{cases}$$

(b) How should $f(3)$ be defined instead to make f continuous at that point?

(3) Give an example of a function with a removable discontinuity (i.e., a hole) at $x = 2$.

Give an example of a function with a jump discontinuity (i.e., a jump or a break forming two separate sections) at $x = 2$.

(4) Let $h(x) = \begin{cases} 3ax + 7, & x < -1 \\ x^2 + a, & x \geq -1 \end{cases}$. Find a so that h will be continuous at $x = -1$. (Use the definition and limits!)

(5) Define $g(-2)$ so that g will be continuous if $g(x) = \dfrac{x^2 - 3x - 10}{x^2 + 2x}$.

(6) Given $f(x) = |x^3 + 2x^2 - 8x|$. Find the values of x where f' is discontinuous. (Note: That's f', not f.)

(7) Let $k(x) = \begin{cases} 3x - 2, & x < 2 \\ 5, & x = 2 \\ x^2, & x > 2 \end{cases}$. Determine, using the definition and limits, whether k is continuous at $x = 2$.

(8) Find each limit. Approximate and guess if necessary.

$\lim\limits_{x \to \infty} \dfrac{\sin x}{x}$ $\lim\limits_{x \to \infty} \dfrac{e^x}{x}$ $\lim\limits_{x \to \infty} \dfrac{\ln x}{x}$