BC 2 Euler Encore 1 Name:________________________

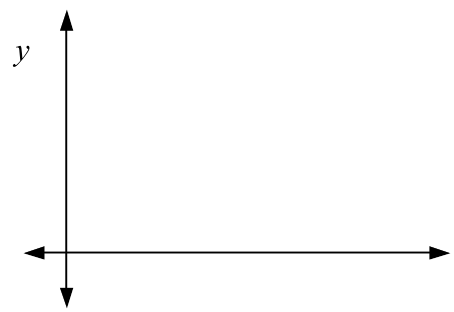
We began last semester with a rate of change function $y'$, which we now know as the derivative function. At that point, we didn't know how to find antiderivatives (or derivatives, for that matter). Instead, we found a piecewise-constant approximation of $y'$ based on the value of $y'$ at the left-hand endpoint of each interval. Using this, we found a continuous, but not differentiable, function $y$ based on our approximation. We now revisit this idea, called Euler's Method.

First, a little review. Recall: new $y$ = old $y$ + rate $\cdot\ \Delta x$

(1) Let $y' = 2x + 1$ on $0 \le x \le 6$ and let $y(0) = 0$. Use stepsize = 2.

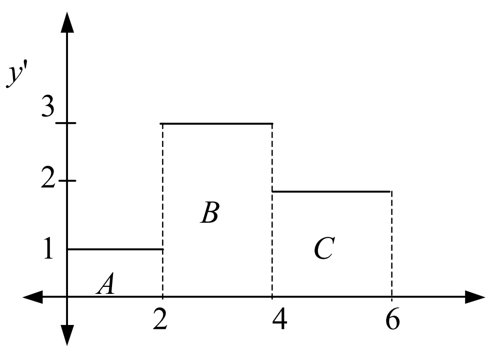
Sketch $y'$ (actual) and $y'$ (approx).

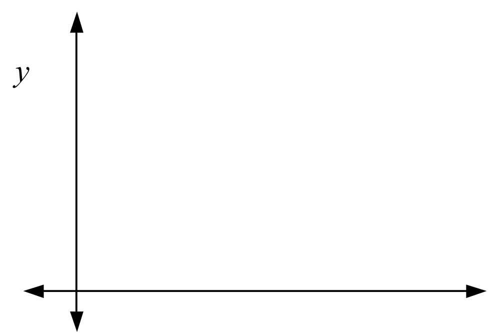

Sketch $y$ based on $y'$ (approx), showing clearly the endpoints of each segment.




(2) Below, $y'$ (approx) is given.

(a) Sketch $y$, assuming that $y$ is continuous and $y(0) = 0$. Mark the endpoints of each segment clearly.




(b) Find the area of each of the following:

rectangle $A$ rect $A$ + rect $B$ rect $A$ + rect $B$ + rect $C$

(c) Explain (clearly and concisely) how $y$ relates to these areas.

(3) Back to $y' = 2x + 1$ on [0, 6] with $y(0) = 0$.

(a) Sketch $y'$ (appprox) with step size = 1 and find $y$, marking the endpoints clearly.

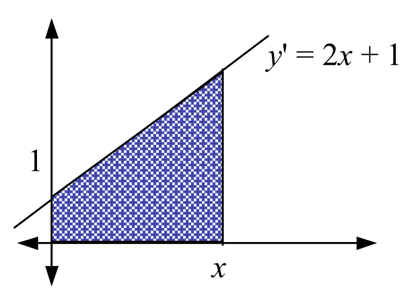
Consider the areas of the 6 rectangles and the sums formed as in (2). Does the same pattern follow?

(4) What should be done to create a better estimate of the area under $y'$?

(5) Evaluate $\int_0^6 (2x+1)\,dx$ exactly. Then use Euler on the calculator with a small step size. Does the program give a good estimate of the actual value of the area?

(6) (a) Find the general antiderivative of $y' = 2x + 1$ and then find the specific one which passes through (0, 0).

(b) Find the area of the trapezoid from 0 to $x$ as shown (in terms of $x$).




(c) Explain clearly what you've discovered on these two pages.