

This activity sheet is designed to find a rule for differentiating the composition of two or more functions. We've already considered several special cases dealing with constants. Here, we try to find a more general rule.

Ask *Mathematica* to find the derivatives of each of the following. Try to determine how each part of the derivative was obtained. The *Mathematica* coding is given to you in several of the first examples. (Use either the carat or the palette for powers.)

$$(1) \quad y = (x + 4)^{10} \quad \Rightarrow \quad y' =$$

Enter: **D[(x + 4)^10, x]**

$$(2) \quad y = (x^2 + 1)^5 \quad \Rightarrow \quad y' =$$

$$(3) \quad y = \ln(x^4 + 2) \quad \Rightarrow \quad y' =$$

Enter: **D[Log[x^4 + 2], x]**

$$(4) \quad y = e^{x^2+1} \quad \Rightarrow \quad y' =$$

Enter: **D[E^(x^2 + 1), x]**

$$(5) \quad y = e^{\tan 3x} \quad \Rightarrow \quad y' =$$

Enter: **D[E^Tan[3x], x]**

$$(6) \quad y = \ln(\sec x) \quad \Rightarrow \quad y' =$$

Enter: **D[Log[Sec[x]], x]**

$$(7) \quad y = \sqrt{x^3 + 1} \quad \Rightarrow \quad y' =$$

$$(8) \quad y = 2^{\sec x} \quad \Rightarrow \quad y' =$$

Watch the brackets on trig functions: for $\sin^3 x + \sin(x^3)$, enter **Sin[x]^3 + Sin[x^3]** and watch the output carefully!

$$(9) \quad y = \sin(x^5) \quad \Rightarrow \quad y' =$$

$$(10) \quad y = \cos(x^3 + 4x + 1)$$

$$(11) \quad y = \cos^7 x$$

$$(12) \quad y = \sin^5(3x)$$

$$(13) \quad y = 5\sin^3(x^4)$$

$$(14) \quad y = \tan^4(2x)$$

Chain Rule: If $k(x) = f(g(x))$, then $k'(x) =$

Now, **without** the computer, use the chain rule to differentiate each of the following functions. Do not simplify.

$$(1) \quad y = (2x + 7)^{14} \quad \Rightarrow \quad y' =$$

$$(2) \quad y = 5(x^3 + 1)^{-2} \quad \Rightarrow \quad y' =$$

$$(3) \quad y = 2(3x^5 - 6x^2 + 18)^{23} \Rightarrow y' =$$

$$(4) \quad y = (x^{1/4} + 3x^{2/3} + 1)^5 \Rightarrow y' =$$

$$(5) \quad y = \ln(4x^2 + 3x - 1) \Rightarrow y' =$$

$$(6) \quad y = (\sin x)^{-4} \quad \Rightarrow \quad y' =$$

$$(7) \quad y = 4 \tan^5 x \quad \Rightarrow \quad y' =$$

$$(8) \quad y = \cos(5x^2 - 3x + 2)$$

$$(9) \quad y = 4\exp(\sqrt{x}) = 4e^{\sqrt{x}}$$

$$(10) \quad y = \frac{(x^2 - 1)^3}{4x + 3}$$

$$(11) \quad y = (5x - 4)^3 \cdot (x^4 + 1)^2$$

$$(12) \quad y = (\ln(6x + 2))^3$$

$$(13) \quad y = \cot^2 x \cdot \tan(4x)$$

$$(14) \quad y = \frac{\sin^3(2x)}{\cos^4(5x)}$$