BC 2 Approximations 1 Name:________________________

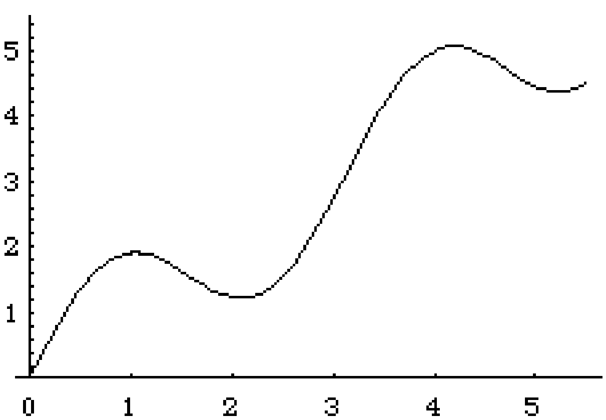
Antiderivatives are great when you can find them, but all too often, that is not possible. We need to study some different ways to approximate integrals. We'll return to the signed area of rectangles we recently studied on the Euler Encore worksheets.

For our example, let $f(x) = x + \sin(2x)$ on [1, 5], and we want to approximate the integral $\int_1^5 (x + \sin(2x))\,dx$.

Our first attempt at an approximation will be a quick, rough estimation. Graph the function on your calculator. Now, have the calculator draw a horizontal line. (On the 89, choose F7, and then Horizontal. On the 83, go to Draw, and then choose Horizontal.) Use the cursors to move the horizontal line up or down until it approximates the average value of the function on the interval [1, 5]. Multiply this y-value (height of the line) by the width of the interval to give the area of the rectangle – the approximate value of the integral. What did you obtain as your estimate?

This is quick and simple, but it may not give a sufficiently accurate result. We begin our search for more accuracy. To obtain an estimate in another manner, we will use more rectangles, just as we used rectangles with Euler's method.

Left-hand Approximation



In this example, we choose $n = 5$ equal subintervals to create a "partition" of the interval [1, 5]. Thus, each will have length $\Delta x = 0.8$. Carefully draw vertical segments at each of the endpoints of all five subintervals. Then create five rectangles using the height of the function at the left-hand endpoint of each subinterval.

Complete the following lines using three decimal places to find the area of the rectangles.

Area (1) + Area(2) + Area(3) + Area(4) + Area(5)

$= f(1)\cdot(0.8) + f(1.8)\cdot(0.8) +$

$= 1.527 +$

$=$

Right-Hand Approximation

We choose again to create five congruent subintervals on the interval [1, 5], each with $\Delta x = 0.8$. Draw the corresponding vertical segments. Create rectangles by using the height of the function at the right-hand endpoint of each subinterval.

Below, create the sums and find the approximation to three decimal places as before.

Midpoint Approximation

Use the same subintervals as before. This time, find the height of each rectangle by using the value of the function at the midpoint of each subinterval. Draw the rectangles carefully, and find the area of each rectangle and the sum as before.

Approximation by Choice

Your choice! We'll continue to use five subintervals to cover the interval [1, 5], but you are to choose the width of each. (They may all be different.) Then choose an arbitrary value of x within each subinterval that will be used to find the height of the function for each rectangle. Express clearly what values you have chosen to form the sum, and then find the area to three decimal places.

Now, find the exact area by using the Fundamental Theorem. Then find a three-decimal place approximation to this value. Compare this to the values found above.