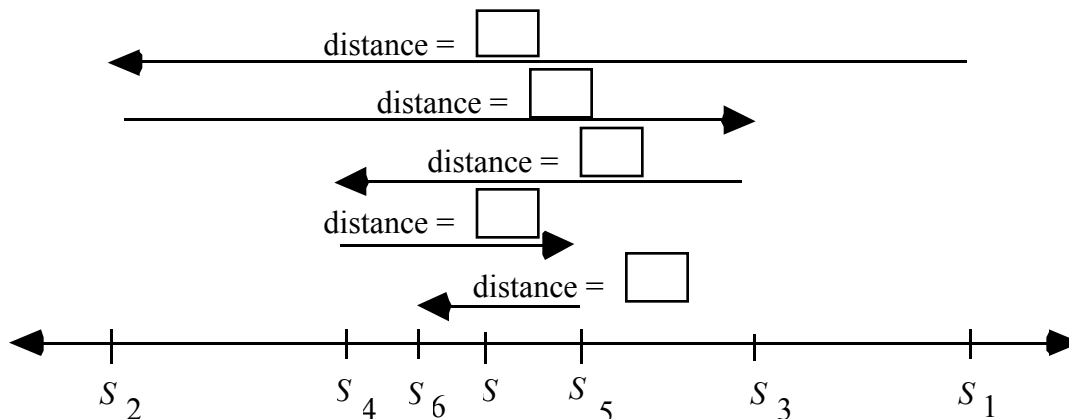


Approximating Alternating Series

Consider the alternating series $a_1 - a_2 + a_3 - a_4 + \dots$ with $a_n \rightarrow 0$. Remember that all $a_n > 0$. Fill in the distances (positive) given by each arrow.

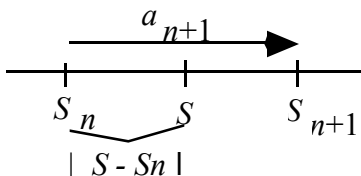


Usually, it is impossible for us to calculate S directly. Therefore, we wish to approximate S by using one of the S_n . For example, let's use S_5 . The next real question or problem is to find out just how good S_5 is as an approximation. In other words, we need to know how close S_5 is to S .

Consider the distance between S_5 and S . Usually, we cannot calculate this distance exactly. Why not? (This may or may not be obvious.)

We can express this distance as $|S - S_5|$. More generally, we need to look at the size of $|S - S_n|$. Since the S_n oscillate in smaller and smaller steps around S , then S is between S_n and S_{n+1} for all n .

Consider this arbitrary stage of the diagram above.



Here, we can see that $|S - S_n| < \underline{\hspace{2cm}}$. More specifically, we have

$|S - S_3| < \underline{\hspace{2cm}}$, $|S - S_5| < \underline{\hspace{2cm}}$, and $|S - S_{20}| < \underline{\hspace{2cm}}$.

(1) Given the alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$,

(a) if we approximate S by using 10 terms, what will be the magnitude of the error? In other words, how large is $|S - S_{10}|$?

Find S_{10} and write an inequality about S .

(b) How many terms are necessary to be sure that the error is less than .01?

(2) Given the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$,

(a) if we approximate S by using 15 terms, what will be the magnitude of the error? In other words, how large is $|S - S_{15}|$?

Find S_{15} and write an inequality about S .

(b) How many terms are necessary to be sure that the error is less than .01?

(3) Given the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$,

(a) if we approximate S by using 19 terms, what will be the magnitude of the error? In other words, how large is $|S - S_{19}|$?

(b) How many terms are necessary to be sure that the error is less than .01?