IMSA

Derivatives of Exponential Functions

Purpose:

To introduce the derivatives of exponential functions through exploration.
Recent changes in calculus programs have emphasized the derivative of exponential functions so that these functions may be used throughout the curriculum rather than waiting until much later. This activity sheet is designed to help develop the rules for the derivatives of functions of the form $y = b^x$ and specifically $y = e^x$. Note that these will not be proved.

Prerequisites:

(1) This activity sheet assumes that students understand the concept of a derivative.
(2) It is assumed that students are familiar with the graphs of exponential functions including the graph of $y = e^x$.
(3) This sheet mentions "nDeriv," referring to a Texas Instruments calculator. Other calculators allowing the graphing of an approximation to the derivative would certainly be usable as well. Students should understand what this command does and how to use it.
(4) It is assumed that students are familiar with basic rules of exponents.

Notes:

This activity sheet is best done in class with heavy intervention from the teacher! Seriously, we have used this repeatedly, and it has been through several revisions. Still, students seem to get stuck at various points and need a bit of pushing. In particular, the bottom of the first page will confuse some students. The top of the second page will be problematic for some as well. Still, we believe in pushing the students through this work, even if we need to help.

This should be followed by clarification of the special pattern for the derivative of $y = e^x$.

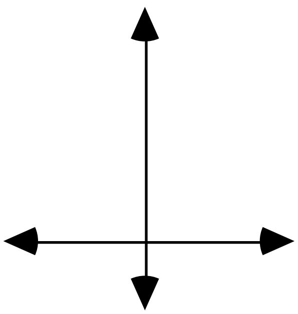
Derivatives of Exponential Functions

Name:______________________

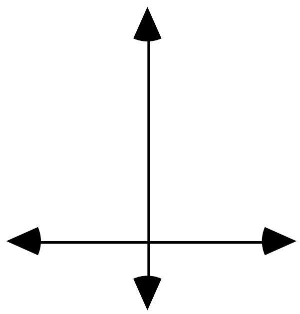
This activity sheet is designed to find the derivative of the function $f(x) = b^x$.

(1) Using nDeriv, graph each function (——) and its derivative (- - -). Try the window [-2, 4] x [-2, 20].

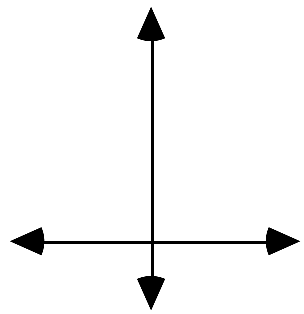
$f(x) = 2^x$



$g(x) = 3^x$



$h(x) = e^x$



(2) What can be said about h and h'? Use the Table to verify this.

(3) To investigate the derivative of $f(x) = b^x$, we consider the ratio of the derivative to the function itself.

We begin with the specific case where b = 2. On the "y=" screen, let Y_1 = 2^X, Y_2 = nDeriv(Y_1, X, X) (TI-83), and $Y_3 = Y_2/Y_1$.

Use "Table." What is true about Y_3? What is its value?

This implies that the derivative of $y = 2^x$ is a multiple of the function itself. This can be seen analytically as well. Using $f(x) = 2^x$, set up the limit to represent the derivative by definition. Simplify a bit by factoring out 2^x. The limit which remains should be a constant. It may be thought of as the derivative of f at what specific value of x? Use nDeriv on the Home screen to verify this.

(4) But what is this constant multiple? While it is a constant for all x in the function $f(x) = 2^x$ as you found above, it is a different constant for each b in the function $f(x) = b^x$. The limit you found in (3) was of the form $\lim_{h \to 0} \frac{b^h - 1}{h}$. Now, b is the variable, and we need to evaluate this limit for different values of b. Choose an appropriately small value of h. On your calculator, plot $Y_1 = \frac{x^h - 1}{h}$, substituting your value of h. What function does this resemble?

In other words, this gives the relationship (b, ________). This means that the ratio of $\frac{\frac{d}{dx}(b^x)}{b^x}$ = ________ , so the derivative of $f(x) = b^x$ is $\frac{d}{dx}\left(b^x\right)$ = ________ .

(5) Find the derivative of each of the following using the rule found above. (For some, you will want to rewrite the expression using properties of exponents.)

$y = 10^x$ $y = 4 \cdot 3^x$ $y = 4^{2x}$

$y = (1/2)^x$ $y = \frac{3}{5^x}$

$y = x^2 + 2^x + 2$ $y = 3 \cdot 6^{-x}$

$y = e^{2x} + x^{2e}$ $y = 3e^{x+2} + 5^{x-1}$