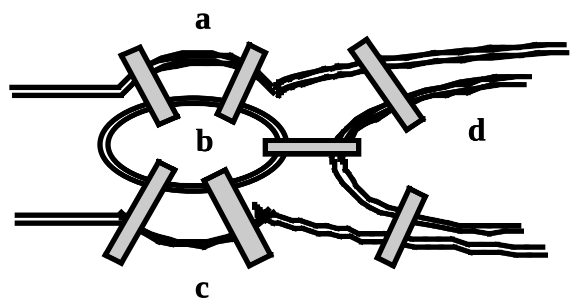
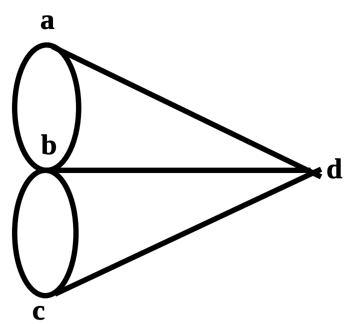
MATHEMATIZATION: A WALK IN THE PARK

by: Susan K. Eddins
Illinois Mathematics and Science Academy

There is a park, *Jardin de la Ville Amélia*, in Barcelona, Spain, where most mornings the older men of the neighborhood meet to walk and visit. While walking in this park with a Spanish colleague during a conference this past fall, I was reminded of Euler's famous *Königsberg Bridge Problem* of 1736 which is often cited as the starting point for the mathematical field know as *graph theory*.



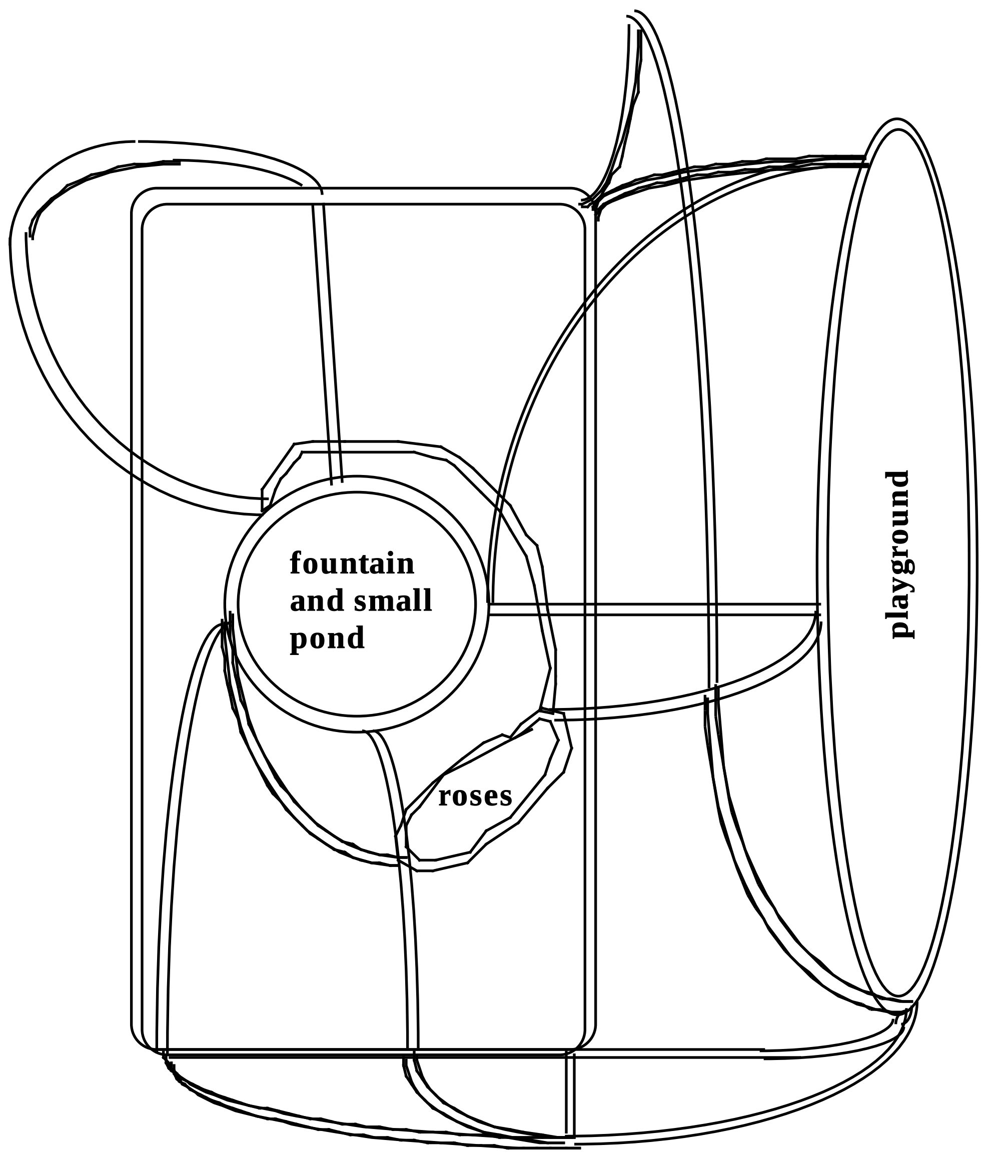
The German town of Königsberg was built on both sides of a river. The town included two islands connected to the shores and each other by a series of seven bridges. The question posed by Euler was whether residents of Königsberg could take a stroll in the evening during which they would cross each of the seven bridges exactly one time. To answer this question, Euler *mathematized* it, that is he took the *essential elements* of the situation and represented them using *mathematical objects*. In this case he used segments of lines and curves and their points of intersection.



Euler chose to represent each land mass by a point, called a *vertex*, and each bridge by a segment, called an *edge*. When mathematized in this way, Euler's question is the same as asking whether it is possible to trace over every segment of this *network* exactly once without lifting your pencil.

Euler reasoned as follows: If the *degree* of a vertex is odd, that is, if it has an odd number of edges coming into it, then you must either *begin* or *end* your tracing at this vertex. To approach a vertex along one edge and leave along another requires a *pair* of edges. Because all four vertices in the Königsberg network are of *odd degree*, traveling across each bridge exactly one time is **not** possible.

The challenge presented to you is to use the drawing of the *Jardin de la Ville Amélia* to mathematize the situation and to determine whether Dr. Perez-Pardo could walk along every path in the park exactly one time during his morning stroll. If it is possible, where must he begin and end? If it is not possible in one walk, how many different walks would it take for him to be able to do so and where could he begin and end each? [Mail you solutions along with your comments back to IMSA.]



Jardin de la Ville Amélia

[Hint: You may need to think carefully about what you represent as vertices.] ✍

• Kenney, Margaret J. (editor), Discrete Mathematics Across the Curriculum, (the 1991 Yearbook of the National Council of Teachers of Mathematics), NCTM, Reston, VA, 1991.