THE "JBTTMH"
(The JB Tate Trigonometric Mnemonic Hexagon)

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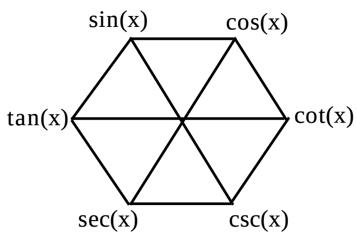
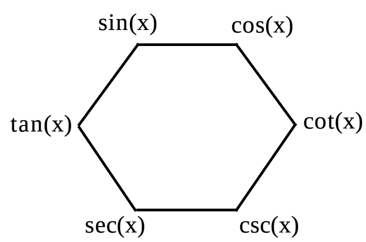
Label the vertices of a regular hexagon in **the following order** with the names of the six trigonometric functions. The *location* of each function can be used to help you remember many trigonometric identities.



1) All the main diagonals of this hexagon are connecting reciprocal functions.
e.g. $\sin(x) = \dfrac{1}{\csc(x)}$, etc . . .



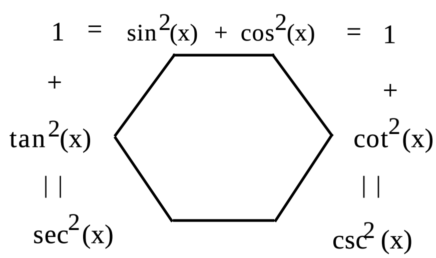
2(a) The function at any vertex is equal to the ratio of the next two consecutive vertices in either direction.
e.g. $\sin(x) = \dfrac{\cos(x)}{\cot(x)}$

$\tan(x) = \dfrac{\sec(x)}{\csc(x)}$

. . . and this works in both directions!

Another way to describe this relationship is:

2(b) The function at any vertex is the product of the functions on either side.
e.g. $\sec(x) = \tan(x) \cdot \csc(x)$.



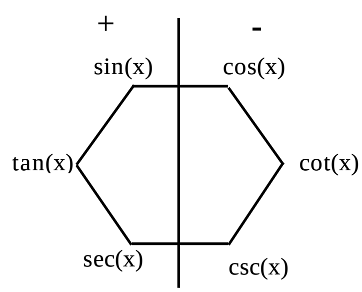
3) The familiar:

$\sin^2(x) + \cos^2(x) = 1$

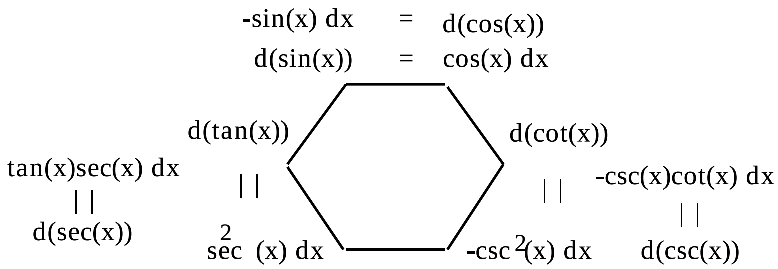
$1 + \tan^2(x) = \sec^2(x)$

$1 + \cot^2(x) = \csc^2(x)$

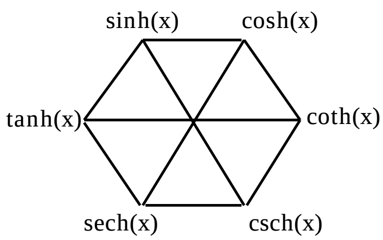
can also be found.



4) Patterns can also be found for the *derivatives* of the trigonometric functions. Sketch in a vertical line separating the hexagon into a left (positive) side and a right (negative) side. The derivative of any function on the left will be positive and the derivative of any function on the right will be negative. That is, the sign of the derivatives achieved using the functions on the left will not change. On the right there is a sign change implied by the negative in the formula.



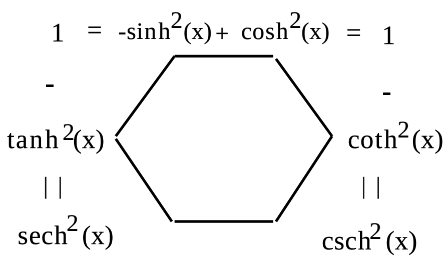
e.g. The derivatives of the sin(x) and the cos(x) are across from one another. The derivatives of tan(x) and cot(x) are "down" and squared. The derivatives of sec(x) and csc(x) "bounce" on sec(x) and csc(x) back up to tan(x) and cot(x).



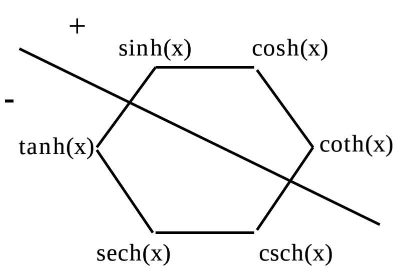
5) This can be extended to the hyperbolic trigonometric functions.

a) The diagonals are reciprocals.

b) The function at any vertex is the product of those functions on either side.



c) $\cosh^2(x) - \sinh^2(x) = 1$
$1 - \tanh^2(x) = \text{sech}^2(x)$
$1 - \coth^2(x) = \text{csch}^2(x)$



d) Their derivatives also appear, but the line separating the hexagon into positive and negative halves is at an angle.

$$d(\sinh(x)) = \cosh(x)\,dx$$
$$d(\cosh(x)) = \sinh(x)\,dx$$
$$d(\coth(x)) = \text{csch}^2(x)\,dx$$
$$d(\tanh(x)) = -\text{sech}^2(x)\,dx$$
$$d(\text{sech}(x)) = -\text{sech}(x)\tanh(x)\,dx$$
$$d(\text{csch}(x)) = -\text{csch}(x)\coth(x)\,dx$$

If you find more relationships, please feel free to share them with us along with your comments on the IMSA Math Journal. We welcome your responses. (For further details please refer to the back side of the front cover for the mailing address.) ✍