APPLYING THE VERTICAL LINE TEST . . . BE CAREFUL!

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There are times when students follow procedures, obtain correct answers and then move on without complete understanding of the concepts at hand. Sometimes the students even leave with incorrect perceptions. As teachers we must take responsibility for such happenings.

For example:

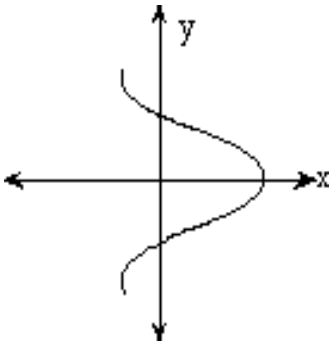


Diagram #1

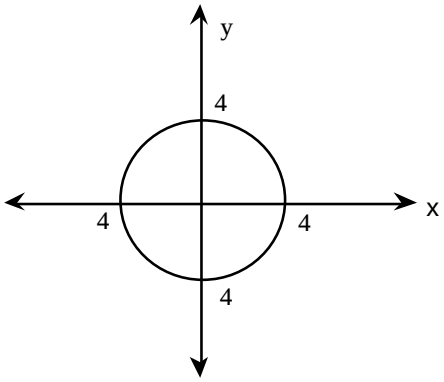


Diagram #2

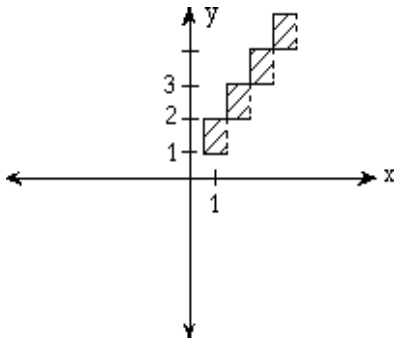


Diagram #3

Which of the relations shown above are functions?

Typically, students will "blindly" follow the vertical line test with little or no real thinking taking place. The students will boldly declare that none are functions since in each graph a vertical line can be drawn that contains more than 1 point of the graph.

Each of the graphs could easily represent a function:

In Diagram #1 -- the graph could represent a function of y. That is, x = f(y). The graph could be interpreted to represent data where each y-value has exactly one x-value associated with it.

In Diagram #2 -- suppose that the circle shown was graphed by using parametric relations $x = 4\cos\mathbf{t}$ and $y = 4\sin\mathbf{t}$. Thus, the graph represents a parametric function where each **t** value produces a unique (x,y) value.

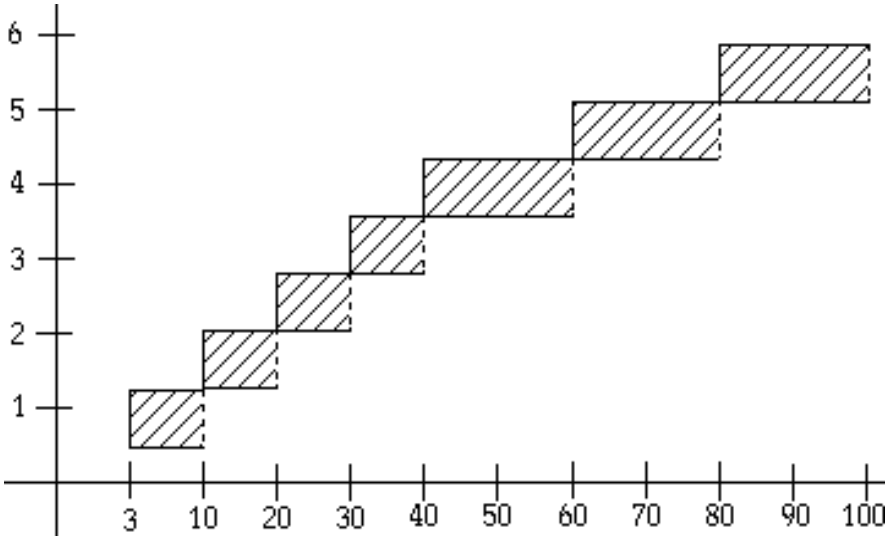
In Diagram #3 -- suppose that each x-value were to map into a closed interval [a,b]; i.e. suppose $\mathbf{f}(x) = [\text{INT}(x),\text{INT}(x + 1)]$. Clearly, each x maps a unique interval, that is $\text{INT}(x) \le y \le \text{INT}(x + 1)$. Hence, **f** is a function.

On the surface we may appear to be too "picky" since many students may never encounter such functions. Even so, we need to think about what we teach, and continually look for real world applications and examples.

In closing, consider data taken from a recent IAMS dog food advertisement.

Weight in Pounds	Cups Per Day for Weight Maintenance
3 to 10	$\frac{3}{4} - 1\frac{1}{4}$
10 to 20	$1 - 2\frac{1}{4}$
20 to 30	$2 - 2\frac{3}{4}$
30 to 40	$2\frac{3}{4} - 3\frac{1}{4}$
40 to 60	$3\frac{1}{4} - 4\frac{1}{4}$
60 to 80	$4\frac{1}{4} - 5\frac{1}{4}$
80 to 100	$5\frac{1}{4} - 6$

The graph for this data is shown below:

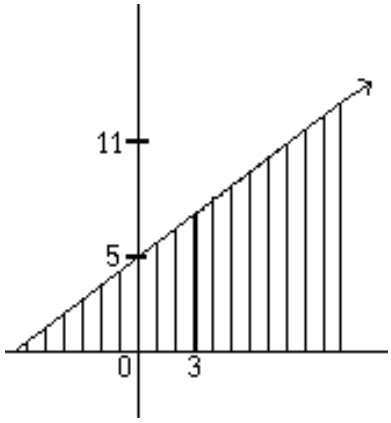


Is the graph a function?

The following example is offered in conjunction with the above article:

$f(x) \le 2x + 5$ for $y \ge 0$ can be thought of as a function of x with intervals as outputs:

x	–2	0	2	4	6	
y	≤ 4	≤ 5	≤ 6	≤ 7	≤ 8	



< Dr. Stanley Hartzler>