

CALCULATOR CURIOSITIES

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The Problem: 45,444,672,000 is the product of the ages of a certain number of teenagers. How could you determine the number of teenagers and their ages?

Our Answer: The object is to use ages that will divide into the original number until it becomes 1. We decided to use our TI-82 calculators to divide by the "teen primes", 13, 17, and 19. We observed the following, repeatedly dividing by 19.

$$\frac{45\,444\,672\,000}{19} = 2\,391\,824\,842 \quad \nearrow \quad \frac{2\,391\,824\,842}{19} = 125\,885\,518 \quad \nearrow \quad \frac{125\,885\,518}{19} = 6\,625\,553.\mathbf{579}$$

(The decimal answer in the third division indicates the original number is only divisible by 19^2 .)

Next we decided to check divisibility by 17.

One of us divided the original number by 17. The other used the result after division by 19^2 .

$$\frac{45\,444\,672\,000}{17} = 2\,673\,216\,000 \qquad \frac{2\,391\,824\,842}{17} = 140\,700.\mathbf{471}$$

In one case it looks like the division yields an integer answer, so the number should be divisible by 17. In the other case a decimal answer indicates the number is NOT divisible by 17. What is going on here? Check this for yourself and try to find out what went wrong.

The explanation:

The TI-82 calculator can only display 10 digits of accuracy on its screen. While 19 is not a divisor of 45 444 672 000, even once, it *appears* to be a divisor, to 10 significant digits, when the division is repeated both a first and second time.

In fact, $45\,444\,672\,000 \div 19 = 2\,391\,824\,842.\mathbf{11}$ and $45\,444\,672\,000 \div 19^2 = 125\,885\,518.\mathbf{006}$

If you realize that 45 444 672 000 is divisible by both 3 and 5, it becomes obvious that 15 divides evenly into the original number, three times.

$$\frac{45\,444\,672\,000}{15^3} = 13\,465\,088$$

Eventually, $45\,444\,672\,000 = 13 \cdot 14 \cdot 15^3 \cdot 16^2 \cdot 17^2$

If you want to try this question on a TI-85, a good number is 5 587 439 130 000. Good luck! 🍀