

AIDS TESTING AND THE USE OF PROBABILITY

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THE SITUATION:

Suppose that the House of Delegates of the American Medical Association has decided to recommend that all physicians be tested for the Human Immunodeficiency Virus (HIV). This is the virus which causes AIDS. They want to include in their recommendation the format (or protocol) that testing should take.

A SEEMINGLY CLEAR-CUT CHOICE:

If two tests are available and what is known is that

Test A is 95% effective in detecting the HI virus when it is present

Test B is 92% effective in detecting the HI virus when it is present

it seems obvious that the House of Delegates would recommend the test which is 95% effective.

Unfortunately, choices about complex issues are rarely that clear-cut. A medical test usually has, not one, but *two* percentages which describe its effectiveness. In fact, the tests above are more completely described by saying that

Test A is 95% effective in detecting the HI virus when it is present but gives a *false positive* result 4% of the time

Test B is 92% effective in detecting the HI virus when it is present but gives a *false positive* result 2% of the time

WHAT "WORKS" REALLY MEANS:

The first percentage in each description above gives the probability of the test **detecting the virus when it is present**. The second percentage gives the probability of the test **returning a positive result when the virus is not really present**, a *false positive*.

For Test A:

If a doctor is HIV+
The probability of a correct diagnosis is 0.95.
The probability of an incorrect diagnosis is 0.05.

If a doctor is HIV-
The probability of a correct diagnosis is 0.96. The probability of an incorrect diagnosis is 0.04.

For Test B:

If a doctor is HIV+
 The probability of a correct diagnosis is 0.92.
 The probability of an incorrect diagnosis is 0.08.

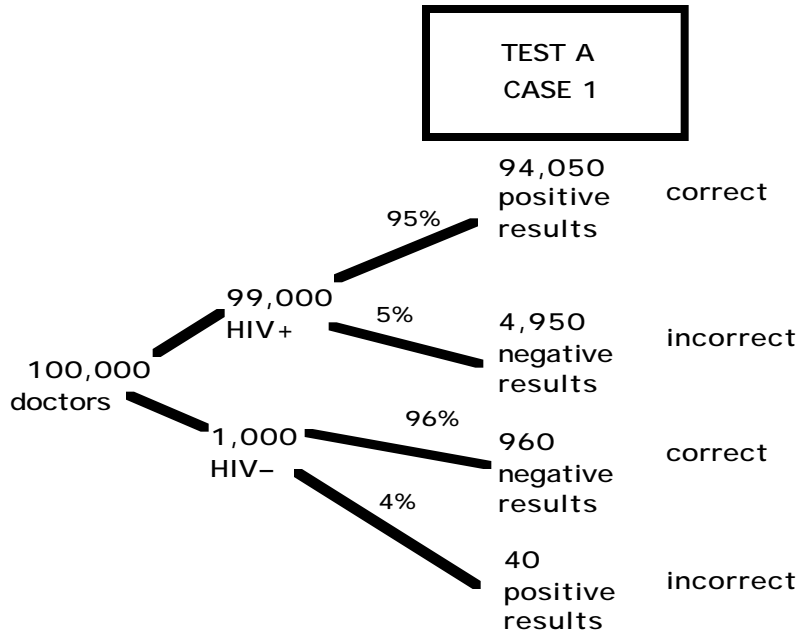
If a doctor is HIV-
 The probability of a correct diagnosis is 0.98. The probability of an incorrect diagnosis is 0.02.

Initial reaction to this question might well be to simply say that there was a 5% error in effectiveness of the first test in detecting the disease when it is present and a 4% error due to false positive for a total 9% error for the first test. Similarly, one could add the 8% and the 2% for the second test to obtain an error value of 10%. Taking these numbers at face value, it appears that the first test is more effective.

To test this conclusion we decided to use a tree diagram to analyze the effect of the two tests on different populations of doctors. We chose a hypothetical population size of 100,000 doctors, and we chose two extreme hypothetical percentages of HIV infection of that population.

Suppose that out of 100,000 doctors, 99%, or 99,000 of the doctors tested, carry the HI virus.

Call this Case 1.



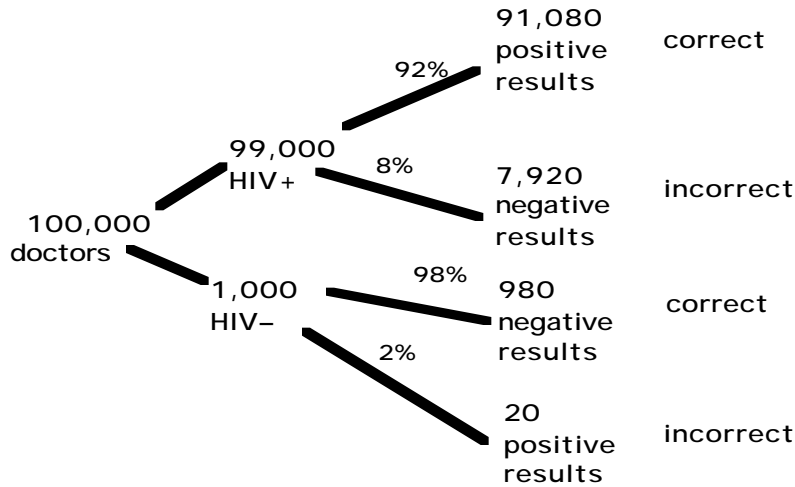
The total number of correct results from Test A Case 1 is: _____.

The total number of incorrect results from Test A Case 1 is: _____.

The probability of a correct result from Test A Case 1 is:

$$\frac{\text{correct results}}{100000} =$$

TEST B
CASE 1



The total number of correct results from Test B Case 1 is: _____.

The total number of incorrect results from Test B Case 1 is: _____.

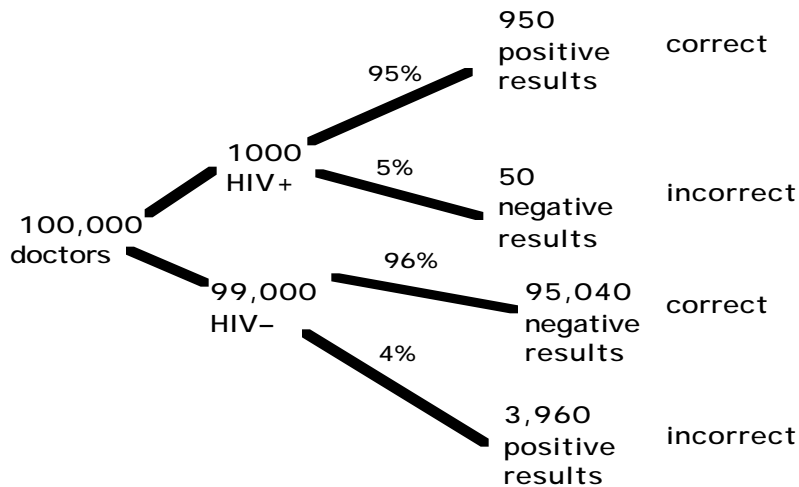
The probability of a correct result from Test B Case 1 is:
 $\frac{\text{correct results}}{100000} =$
 _____.

The scenario that 99% of doctors are HIV+ seems highly unlikely.

Suppose that out of 100,000 doctors, only 1%, or 1,000 of the doctors tested, carry the HI virus.

Call this Case 2.

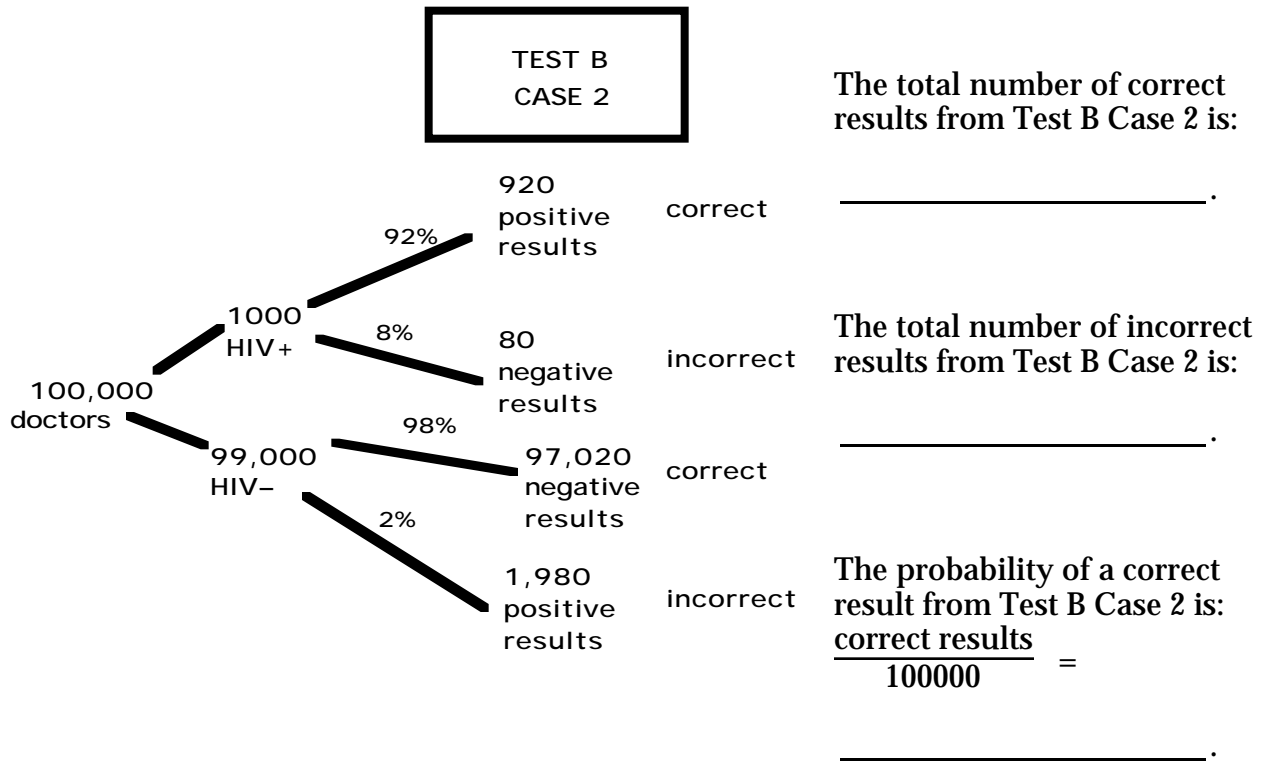
TEST A
CASE 2



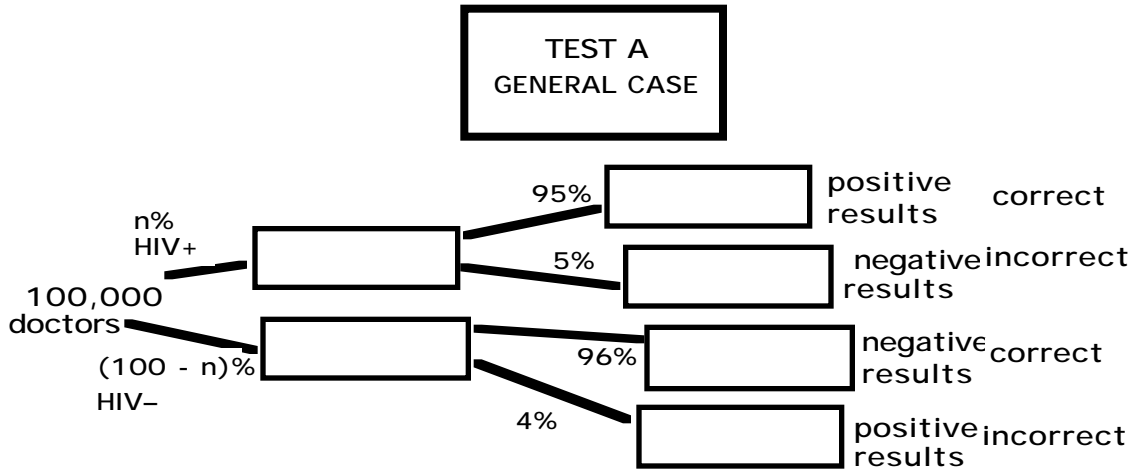
The total number of correct results from Test A Case 2 is: _____.

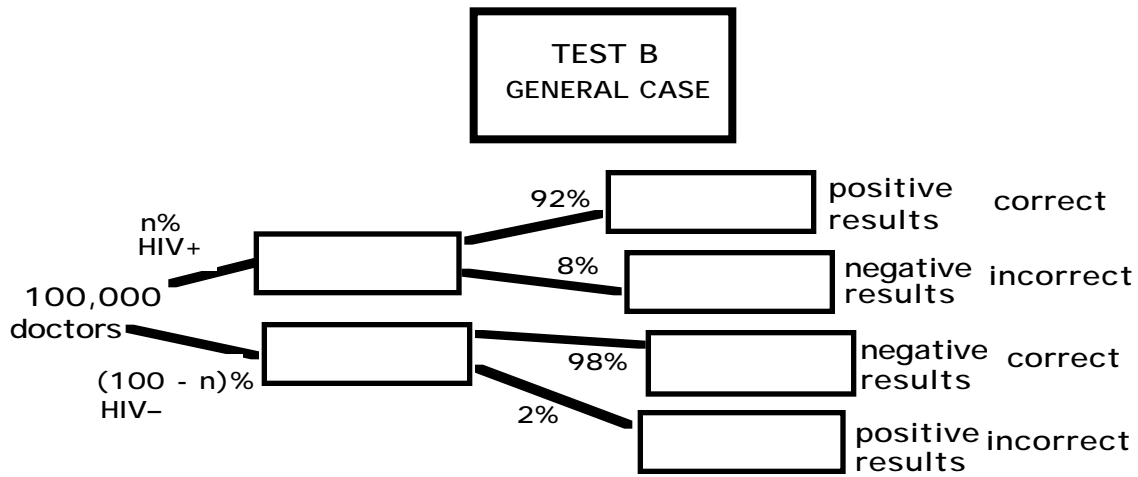
The total number of incorrect results from Test A Case 2 is: _____.

The probability of a correct result from Test A Case 2 is:
 $\frac{\text{correct results}}{100000} =$
 _____.



You probably have noticed that in Case 1, Test A is slightly more reliable while in Case 2, Test B is the more accurate test. Somewhere in between 99% HIV+ and 1% HIV+ is a population of doctors for which either test is equally effective. Complete the tree diagrams below, set the probability of correct results equal to one another, and solve for n to determine that percent.





For a population of doctors that is _____% HIV+ Test A and Test B are equally correct.

In deciding on your recommendation to the AMA House of Delegates, you may want to include some *ethical* considerations as well as strictly mathematical ones. Is it more serious, in your opinion, for the test to fail to identify doctors who carry the HI virus or to identify as positive those doctors who do not carry the virus?

It is possible that you may want to recommend repeated testing. Doctors who test positive with one test, might be required to be retested. All doctors could be tested twice; in this case which test would be the best? Would both tests have to be positive or is one sufficient to designate a doctor as a carrier of the HI virus? Does it matter which test is given first and which is given as a follow-up?

You might want to make tree diagrams for several more scenarios to test your responses. 🖍️