

"QUICKIES"

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The following problems have something in common: each can be solved in less than three minutes. Give yourself fifteen minutes for all of them.

1° For how many positive integers, n , is $n^2 + n + 2$ an integer square?

2° Let m and n be distinct positive integers. Write $m^6 + n^6$ as a sum of two squares, different than m^6 and n^6 .

3° Solve the system:
$$\begin{aligned}x^2 - xy &= a \\ y^2 - xy &= a(a - 1)\end{aligned}$$

4° Let f be a real function such that

$$3f(n - 2) + 2f(3 - n) = n + (-1)^n$$

for all integers, n . Find $f(1993)$.

5° The bases of a trapezoid measure 19 and 93 inches and the other two sides 24 and 70 inches. Find the angle formed by the extended non-parallel sides.