

## VISUAL PATTERNING

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Below is a visual sequence that motivates the relation:

$$(1 + 2 + 3 + \dots + n)^2 = 1^3 + 2^3 + 3^3 + \dots + n^3$$

described in the article "[Exercising With The Math Induction Principle](#)".

x	1
1	1

 $(1)^2 = 1^3$

x	1	2
1	1	2
2	2	4

$$\begin{aligned} (1 + 2)^2 &= 1 + (2 + 4 + 2) \\ &= 1 + 8 \\ &= 1^3 + 2^3 \end{aligned}$$

x	1	2	3
1	1	2	3
2	2	4	6
3	3	6	9

$$\begin{aligned} (1 + 2 + 3)^2 &= 1 + (2 + 4 + 2) + (3 + 6 + 9 + 6 + 3) \\ &= 1 + 8 + 27 \\ &= 1^3 + 2^3 + 3^3. \end{aligned}$$

⋮  
⋮  
⋮

x	1	2	3	...	n
1	1	2	3	...	n
2	2	4	6	...	2n
3	3	6	9	...	3n
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
n	n	2n	3n	...	n <sup>2</sup>

$$\begin{aligned} (1 + 2 + 3 + \dots + n)^2 &= 1 + (2 + 4 + 2) + (3 + 6 + 9 + 6 + 3) \\ &+ \dots + (n + 2n + 3n + \dots + n^2 + \dots + 3n + 2n + n) \\ &= 1 + 8 + 27 + \dots + n^3. \end{aligned}$$