

"QUICKIES"

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The following problems have something in common: each can be solved in less than two minutes. Give yourself ten minutes for all of them.

1° Fill out the blanks, given that the sum of the numbers in any three adjacent spots is equal to 10:

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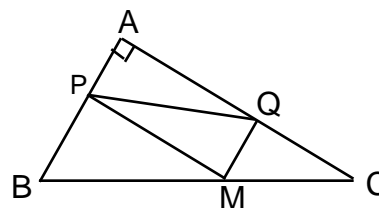
2° Find the greatest multiple of 8 with all the digits different.

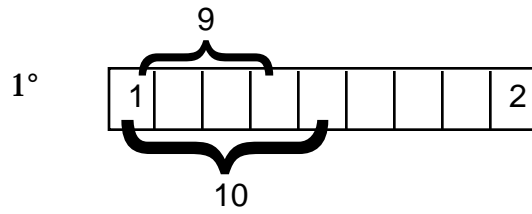
3° Determine the prime numbers x , y , z such that:

$$2x + 3y + 4z = 222 .$$

4° Compute: $(1 - \tan 1^\circ)(1 - \tan 2^\circ) \cdot \dots \cdot (1 - \tan 89^\circ)$.

5° Let M be a mobile point on hypotenuse BC of the right triangle ABC and P , Q the feet of the perpendicular segments from M to AB and AC . For what position of M does the length of PQ reach its minimum?



Hints:

- 2° The greatest number with all the digits different is **9876543210** (unfortunately, it is not divisible by 8).
- 3° The numbers $2x$, $4z$, and 222 are even, therefore $3y$ has to be even.
- 4° Examine the factor in the middle.
- 5° Draw AM.