

## PROPOSED PROBLEMS

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This section contains problems that address high school students (and their teachers!). Both are challenged to either send in solutions to the published problems or to submit new problems.

P-1. Solve the equation:

$$(x + 3)^{1/2} - (x + 2)^{1/3} = 1.$$

P-2. The real numbers  $a, b, c, d$ , belonging to the interval  $[0,1]$ , are displayed in strictly increasing order. Find the minimum possible value of:

$$\frac{1}{a - b} + \frac{1}{b - c} + \frac{1}{c - d}$$

P-3. Solve the system:

$$x + y = \sqrt{4z - 1}$$

$$z + x = \sqrt{4y - 1}$$

$$y + z = \sqrt{4x - 1} .$$

P-4. Let  $\binom{m}{k}$  represent the combinations of  $m$  objects taken  $k$  at a time. Prove that if  $n$  is odd, then the sequence:

$$\binom{n}{1}, \binom{n}{2}, \dots, \binom{n}{\frac{n-1}{2}}$$

contains an odd number of odd numbers.

P-5. Demonstrate that if  $a$  and  $b$  are acute angles, then

$$\frac{\sin^2 a}{\sin b} + \frac{\cos^2 a}{\cos b} = 1$$

if and only if  $a = b$ .

P-6. The diagonals AC and BD of the quadrilateral ABCD intersect at P. Prove that if the sum of the areas of triangles ABP and CDP equals the sum of the areas of triangles BCP and DAP then at least one of the diagonals AC and BD has P as its midpoint.

### Fun Problem

P-7. Inserting symbols other than digits and letters, transform

$$2 = 0$$

into a true equality.

Solutions to these problems should be sent at the address given on the inside front cover. In order to be considered for publication they must be received before January 15, 1993. A publishable solution should be, first of all, correct and complete. Elegant and ingenious solutions are preferred.

Extensions, generalizations, and any other comments are welcome. Please type or print clearly and don't forget to include your name and mailing address with each solved problem.

Proposed problems should be original and be accompanied by complete solutions. Please include appropriate reference and any other materials which may be useful for the editor. Since submissions will not be returned, keep a copy for yourself. 📧