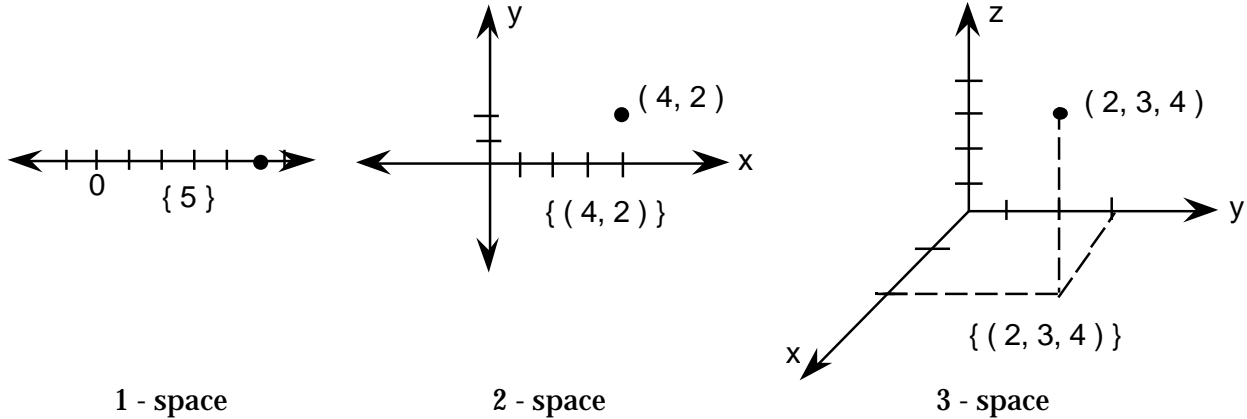


Using Parallel Axes To Discover Mathematical Interconnections

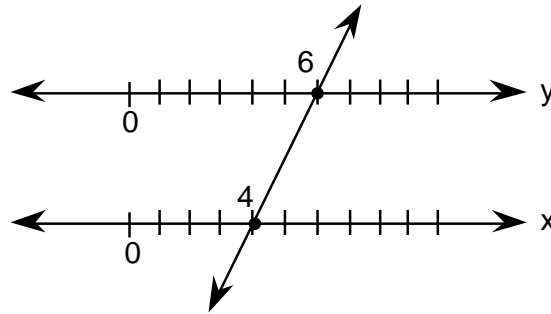
by Charles Hamberg

Illinois Mathematics and Science Academy

In beginning algebra classes students graph "points" using coordinates. Standard ways of representing points in 1, 2, and 3-dimensional space are illustrated.



It is interesting in 2-space to think of the axes as being parallel. In the diagram below, the ordered pair (4, 6) is graphed on parallel axes. Observe that the "point" (4,6) is actually displayed as a line.



We observe that the "point" (4, 6) is actually displayed as a line.

There are many discoveries that students can make by graphing functions on parallel axes.

Consider the graph of $y = 2x - 5$ on both rectangular and parallel axes.

x	y
0	-5
2	-1
5	5

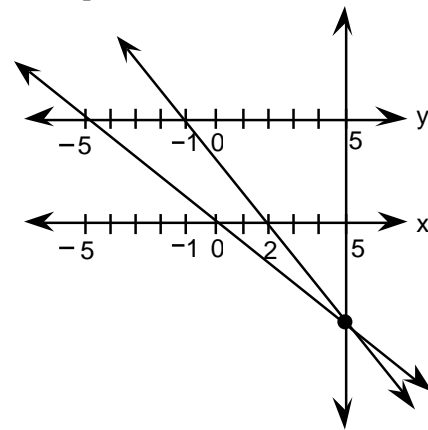
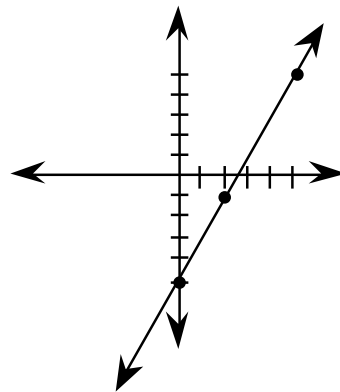


Table of Values

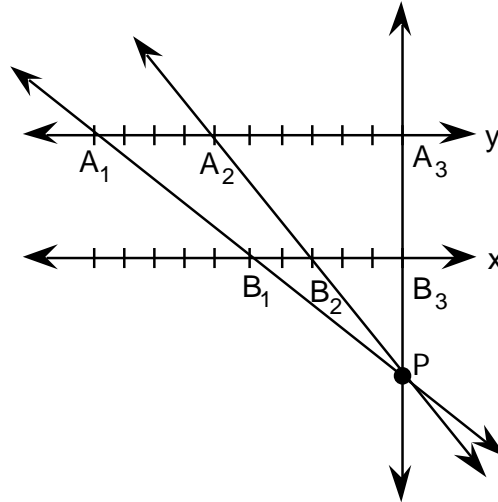
Rectangular Axes

Parallel Axes

On rectangular axes the ordered pairs represent points which determine a line. On parallel axes the ordered pairs represent lines which intersect at a single point. We will call this point the "slope point" of the function $y = 2x - 5$.

We can show by using similar triangles that

$\frac{A_1P}{B_1P} = \frac{A_2P}{B_2P} = \frac{A_3P}{B_3P} = 2$ where 2 is the slope of the function $y = 2x - 5$.



Exploration #1:

Graph the following on parallel axes and observe the location of the slope point with respect to the x and y axes.

- | | |
|----------------------------|----------------|
| 1. $y = \frac{2}{3}x + 2$ | 4. $y = 5$ |
| 2. $y = -4x + 1$ | 5. $x = 4$ |
| 3. $y = -\frac{1}{2}x + 2$ | 6. $y = x + 3$ |

Exploration #2:

- Graph the functions $y = 2x + 3$ and $y = -x + 6$ on the same set of parallel axes. Label the slope points P_1 and P_2 .
- Draw the line P_1P_2 intersecting the x and y axes at x_1 and y_1 .
- Solve the system $\begin{cases} y = 2x + 3 \\ y = -x + 6 \end{cases}$. What do you observe?

Exploration #3:

Repeat exploration #2 for functions:

- $y = 2x + 5$ and $y = 2x - 7$
- $y = x + 3$ and $y = 2x - 4$

You are encouraged to explore other aspects of graphing using parallel axes. In future issues we will expand upon this topic using student discoveries. 📐