

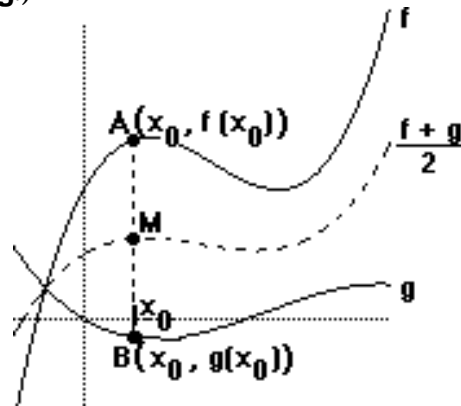
"ADDING" GRAPHS BY AVERAGING AND DOUBLING

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The diagram below shows the graphs of $y = f(x)$, $y = g(x)$, and $y = \frac{f(x) + g(x)}{2}$

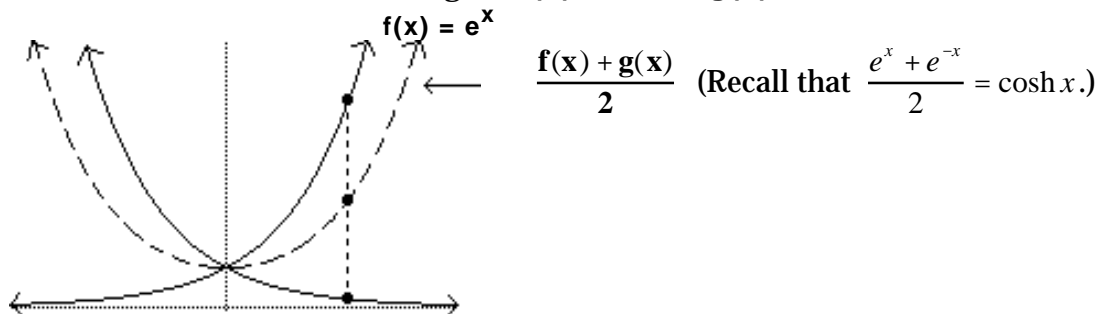
(the arithmetic average of f and g .)



Close inspection reveals that for a given x_0 common to the domains of f and g , M is the midpoint of segment AB .

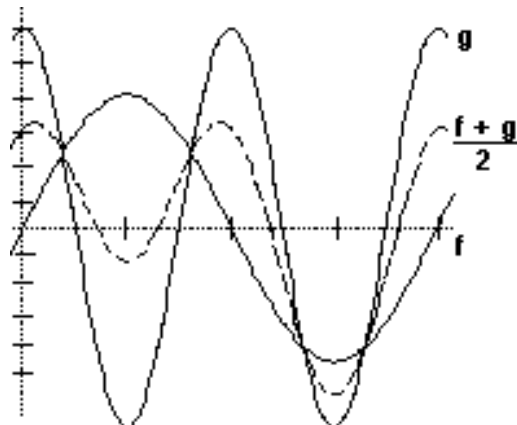
Example 1:

Sketch the arithmetic average of $f(x) = e^x$ and $g(x) = e^{-x}$.



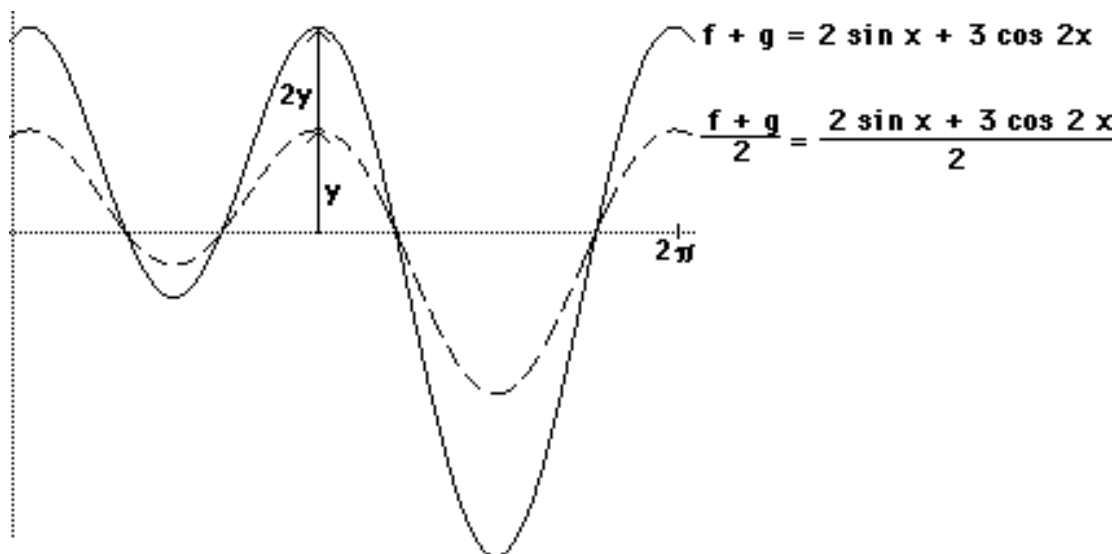
Example 2:

Sketch the arithmetic average of $f(x) = 2 \sin x$ and $g(x) = 3 \cos 2x$ for $x \in [0, 2\pi]$



Suppose we multiply the arithmetic average of f and g by 2.

Graphically, we have:

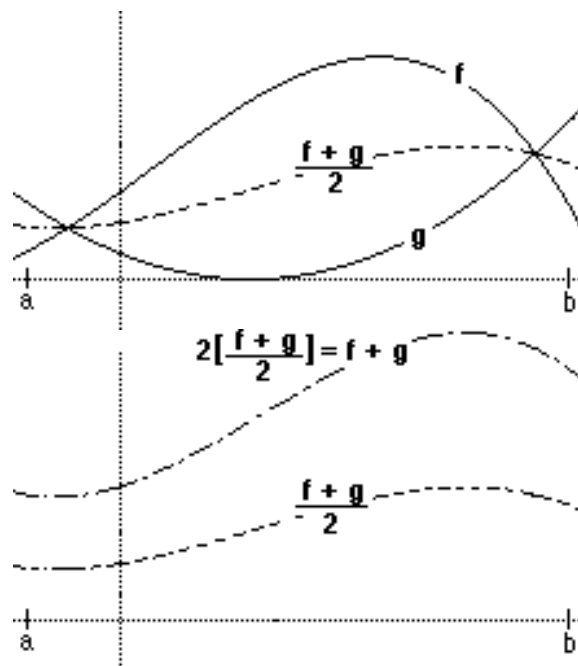


Thus, we see that twice the arithmetic average is the the sum of the two functions f and g .

We now have an alternative method to ordinate addition for adding the graphs of two functions. We summarize:

To graphically add two functions f and g .

- Sketch the graphs of f and g for
- Sketch the arithmetic average of f and g .
- Double the average function.



Teachers incorporating the TI-81 into their teaching could use the TI-81 to visually demonstrate the process by doing the following:

$$\begin{aligned} \text{Set } y_1 &= f(x) \\ y_2 &= g(x) \\ y_3 &= (f(x) + g(x))/2 \\ y_4 &= 2 * ((f(x) + g(x))/2) \end{aligned}$$

Graph the functions: y_1 , y_2 , y_3 and y_4 .

Exploration 1:

Draw the graphs of the arithmetic average of f and g and the sum $f + g$ of f and g for the following:

- $f(x) = 3 \cos 2x$ and $g(x) = -4 \sin x$, $0 \leq x \leq 2\pi$.
- $f(x) = 2^x - 3$ and $g(x) = 2^{-x} - 3$, $-1 \leq x \leq 4$.
- $f(x) = [x]$ and $g(x) = x$, $-5 < x < 5$.
- $f(x) = |x|$ and $g(x) = -x$, $-5 \leq x \leq 5$.
- $f(x) = x$ and $g(x) = 2 \cos x$, $0 \leq x \leq 2\pi$.
- $f(x) = \sin x + \cos x$ and $g(x) = \sin x - \cos x$, $0 \leq x \leq 2\pi$.

Exploration 2:

See if you can apply the technique in a polar system to obtain $r = f(\theta) + g(\theta)$, $0 \leq \theta \leq 2\pi$, where

- $r = f(\theta) = 3$ and $r = g(\theta) = 3 - 2 \sin \theta$.
- $r = f(\theta) = 1 + \sin \theta$ and $r = g(\theta) = 1 - \sin \theta$.
- $r = \sin \theta$ and $r = \cos \theta$.
- $r = \sin^2 \theta$ and $r = \cos^2 \theta$.
- $r = 3$ and $r = \sec \theta$.
- $r = -2$ and $r = \sin \theta$. 🐶